# 2.5 — Short Run Profit Maximization

## ECON 306 • Microeconomic Analysis • Spring 2022

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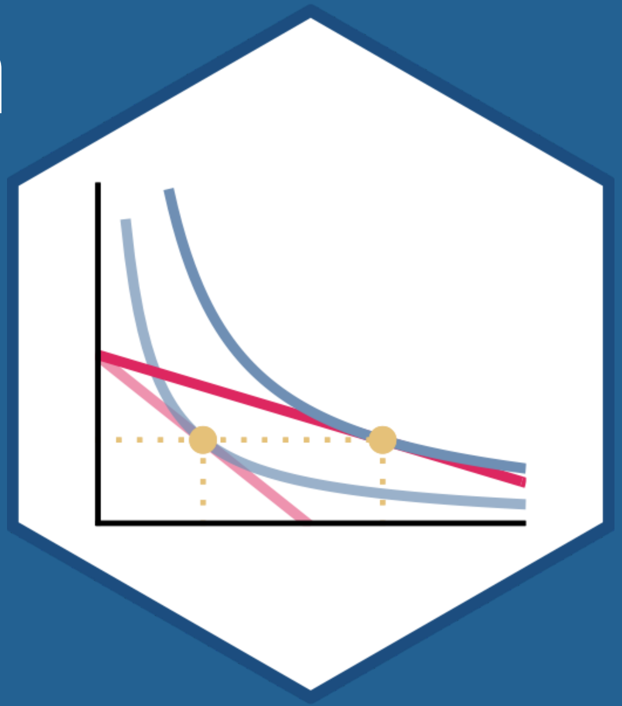
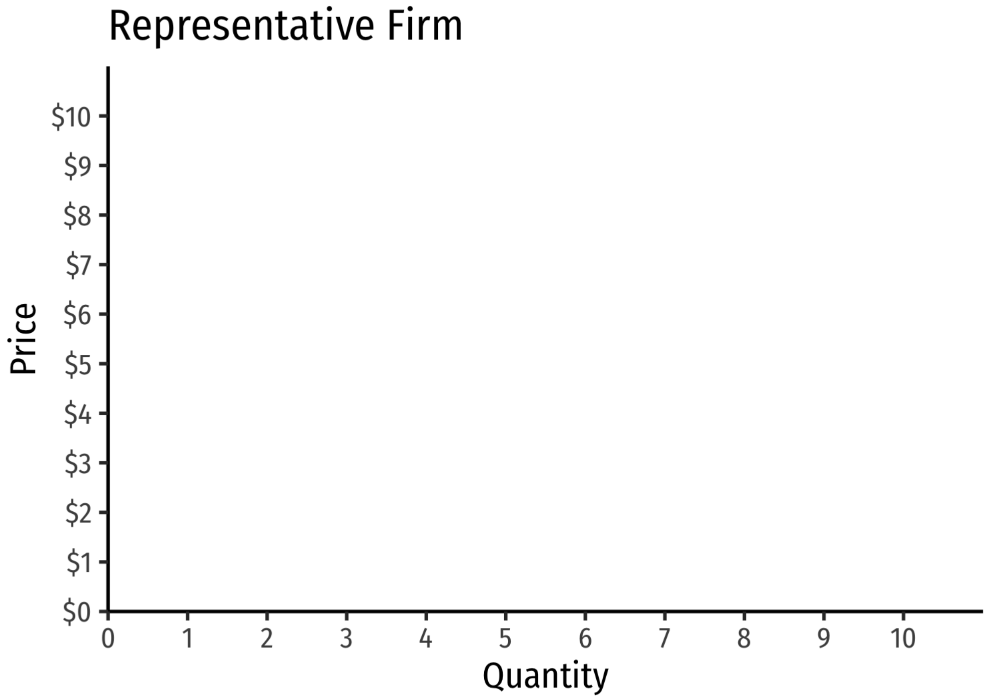
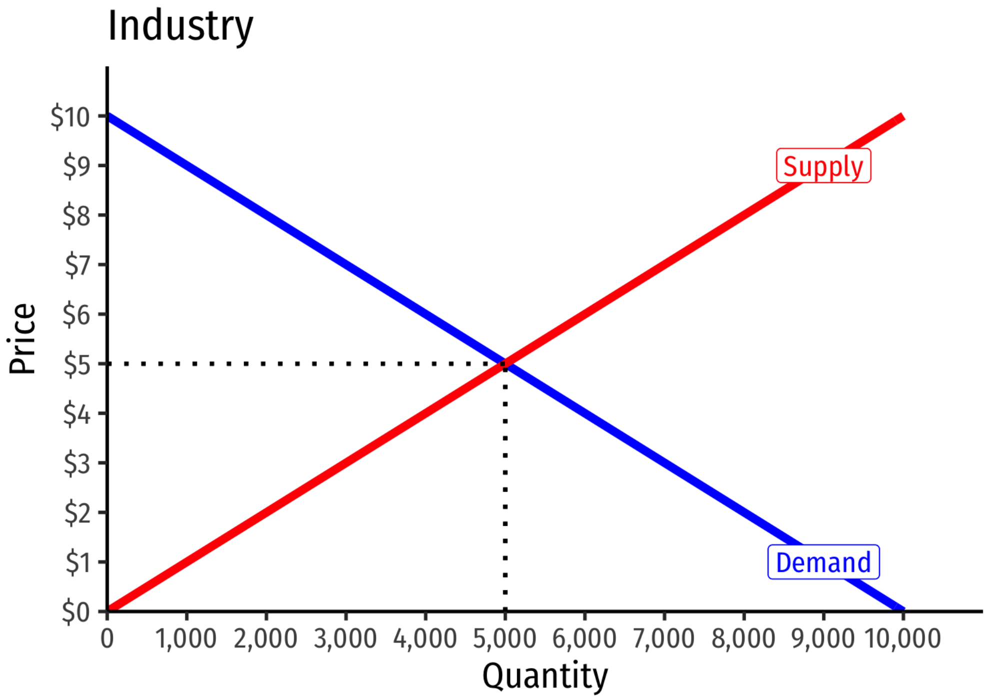


# Outline

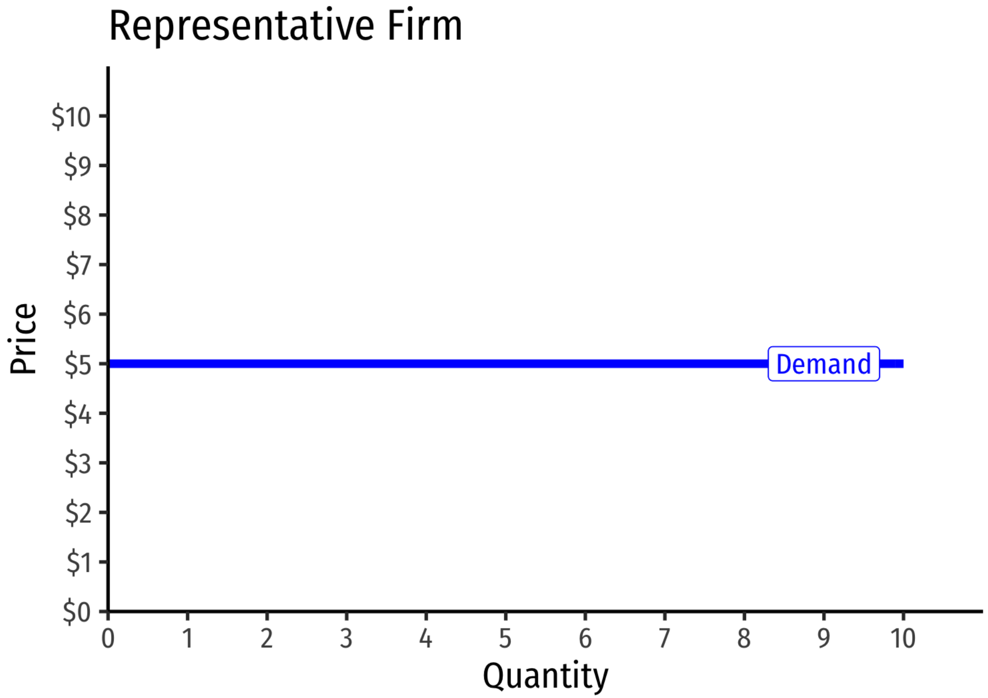
# Revenues

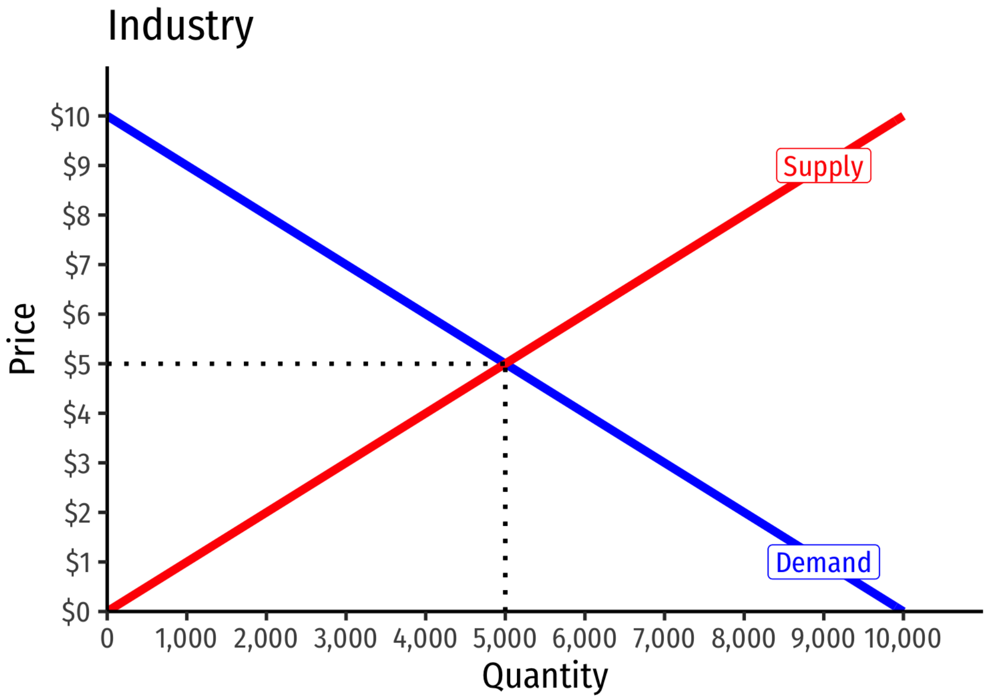
# Revenues for Firms in *Competitive* Industries I



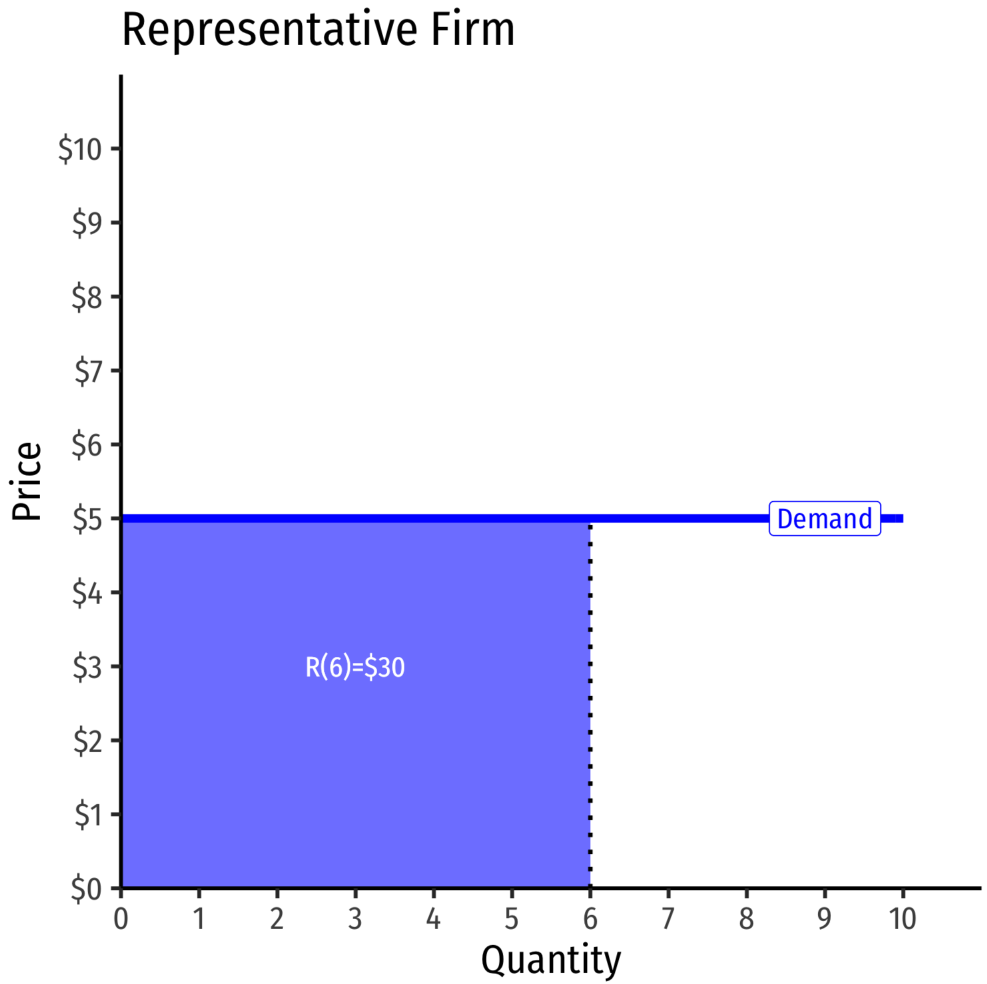
# Revenues for Firms in *Competitive* Industries I




- Demand for a firm's product is **perfectly elastic** at the market price
- Where did the supply curve come from? You'll know today

# Revenues for Firms in *Competitive* Industries II




- **Total Revenue** $R(q) = pq$

# Average and Marginal Revenues

- **Average Revenue**: revenue per unit of output

$$AR(q) = \frac{R}{q}$$

  - $AR(q)$ is **by definition** equal to the price! (Why?)

- **Marginal Revenue**: change in revenues for each additional unit of output sold:

$$MR(q) = \frac{\Delta R(q)}{\Delta q} \approx \frac{R_2 - R_1}{q_2 - q_1}$$

  - Calculus: first derivative of the revenues function
  - **For a *competitive* firm (only), MR(q) = p, i.e. the price!**

# Average and Marginal Revenues: Example

**Example**: A firm sells bushels of wheat in a very competitive market. The current market price is $10/bushel.

For the 1st bushel sold:

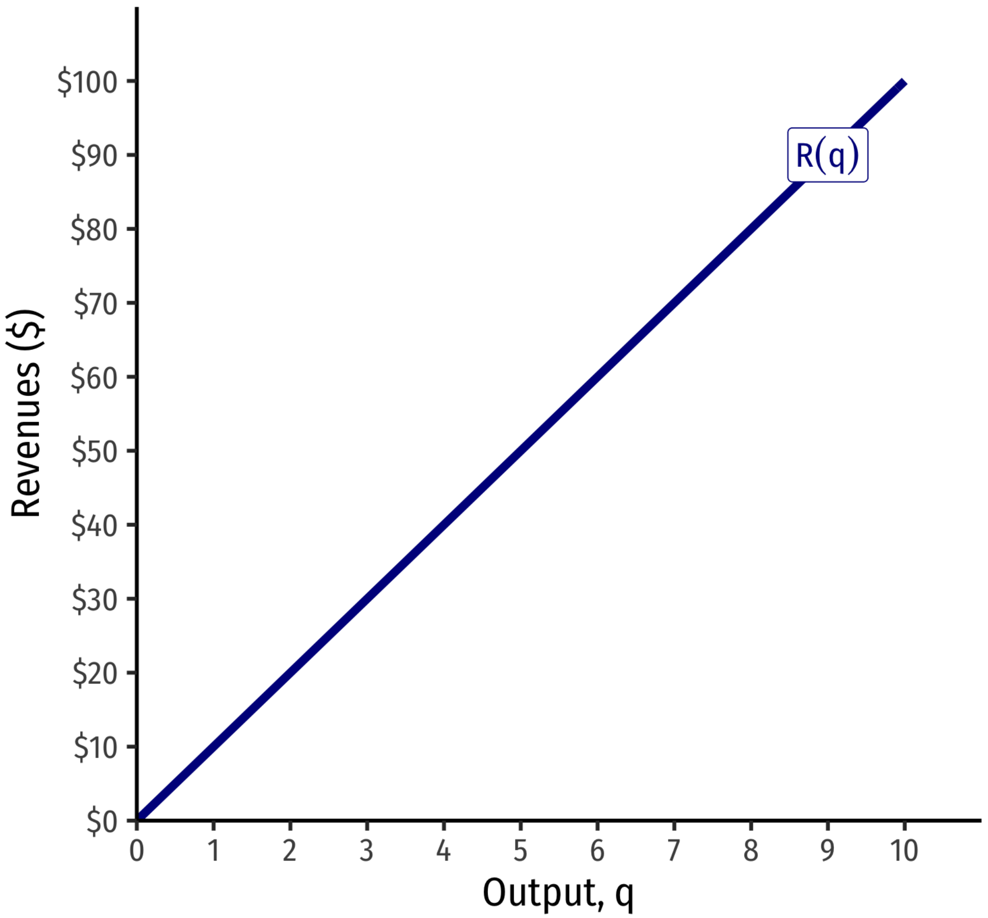
- What is the total revenue?
- What is the average revenue?

For the 2nd bushel sold:

- What is the total revenue?
- What is the average revenue?
- What is the marginal revenue?

# Total Revenue, Example: Visualized

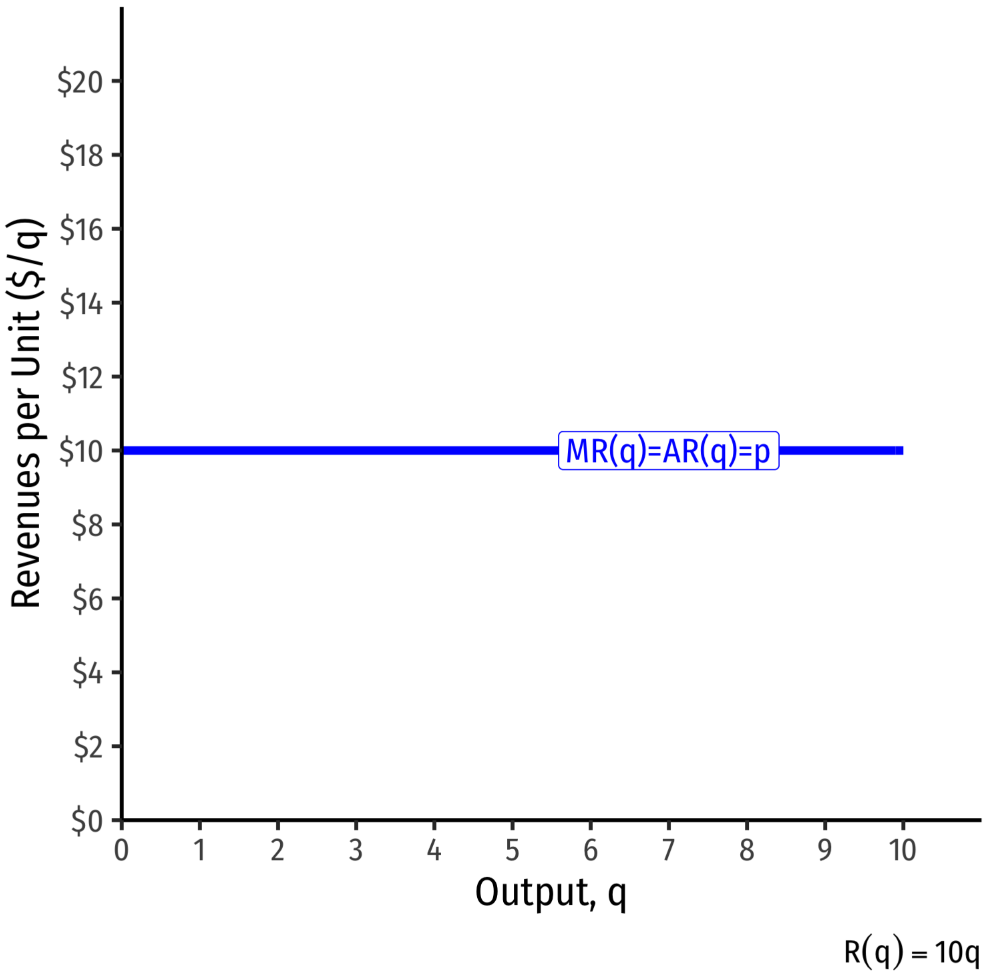
| $q$ | $R(q)$ |
|---|---|
| 0 | 0 |
| 1 | 10 |
| 2 | 20 |
| 3 | 30 |
| 4 | 40 |
| 5 | 50 |
| 6 | 60 |
| 7 | 70 |
| 8 | 80 |
| 9 | 90 |
| 10 | 100 |




R(q) = 10q

# Average and Marginal Revenue, Example: Visualized

| $q$ | $R(q)$ | $AR(q)$ | $MR(q)$ |
|---|---|---|---|
| 0 | 0 | – | – |
| 1 | 10 | 10 | 10 |
| 2 | 20 | 10 | 10 |
| 3 | 30 | 10 | 10 |
| 4 | 40 | 10 | 10 |
| 5 | 50 | 10 | 10 |
| 6 | 60 | 10 | 10 |
| 7 | 70 | 10 | 10 |
| 8 | 80 | 10 | 10 |
| 9 | 90 | 10 | 10 |
| 10 | 100 | 10 | 10 |



# Profits

# Recall: The Firm's Two Problems

1$^{st}$ Stage: firm's profit maximization problem:

1. **Choose:** < output >

2. **In order to maximize:** < profits >

2$^{nd}$ Stage: firm's cost minimization problem:

1. **Choose:** < inputs >

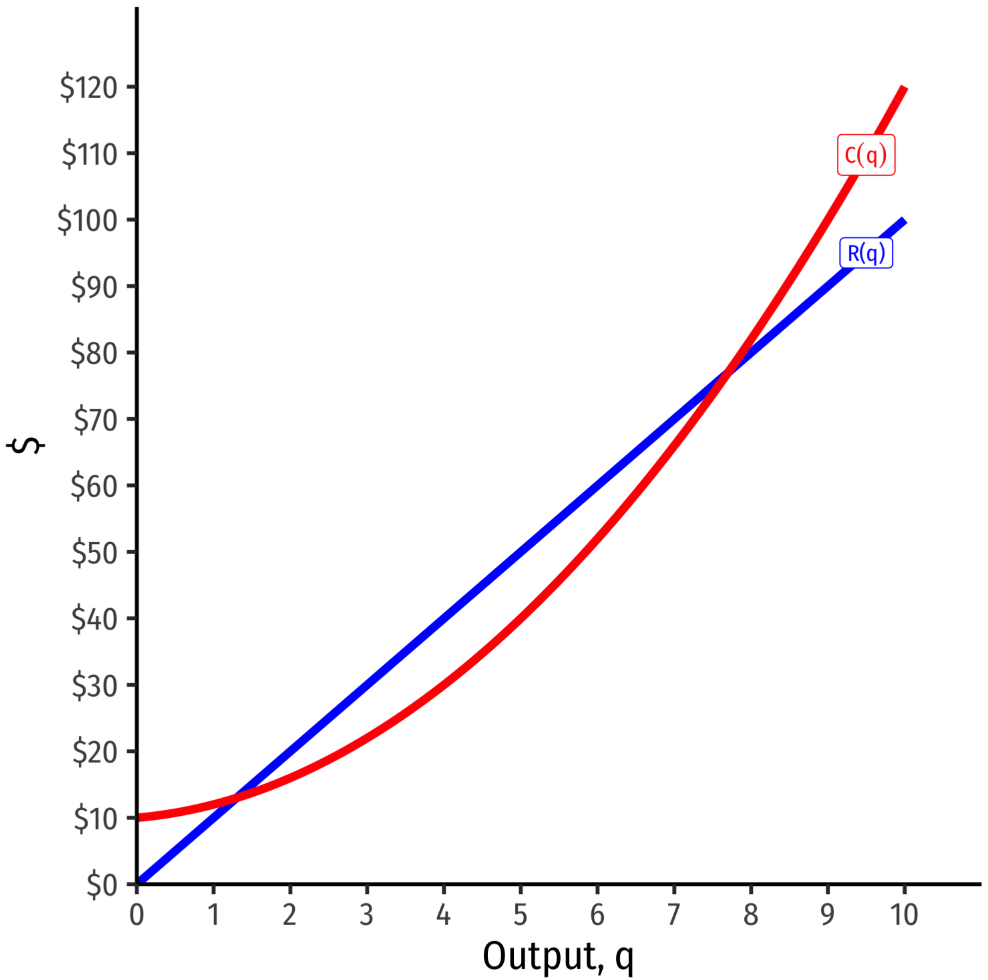
2. **In order to *minimize*:** < cost >

3. **Subject to:** < producing the optimal output >

- Minimizing costs $\iff$ maximizing profits

# Visualizing Total Profit As $R(q) - C(q)$

- $\pi(q) = R(q) - C(q)$



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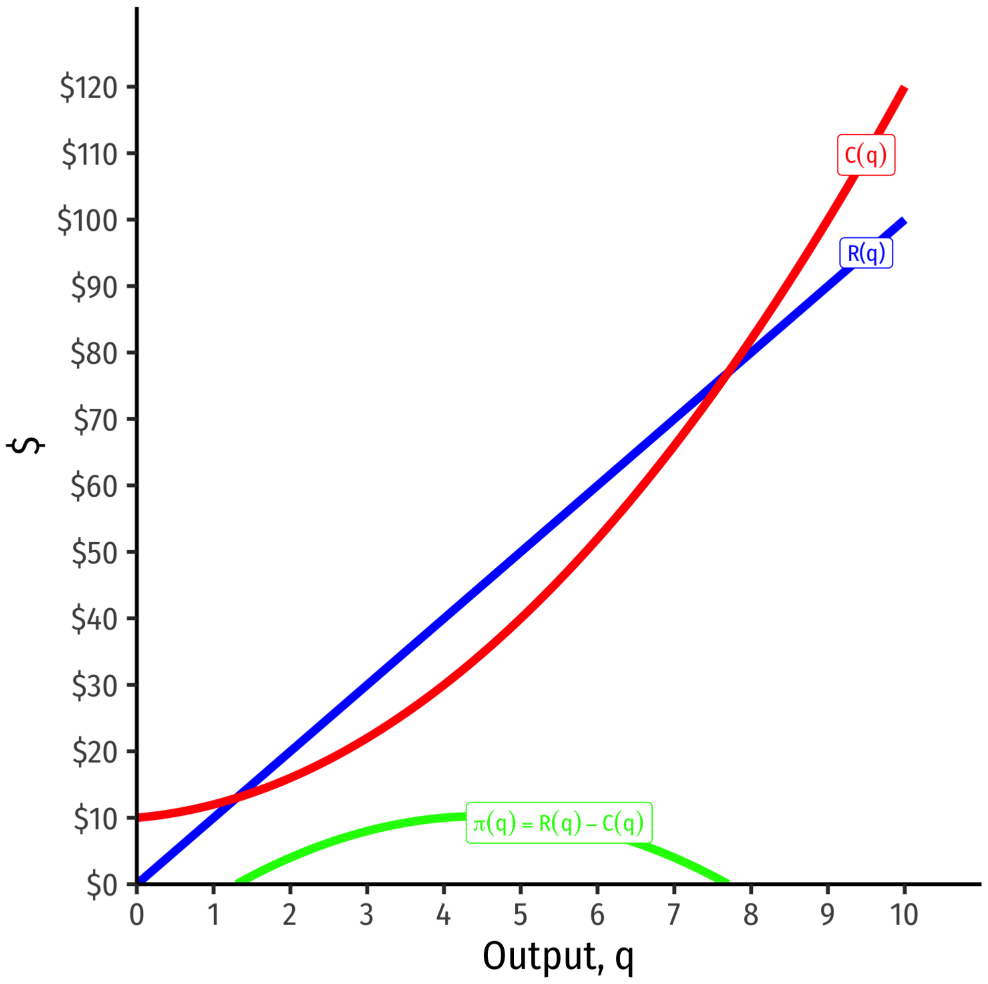
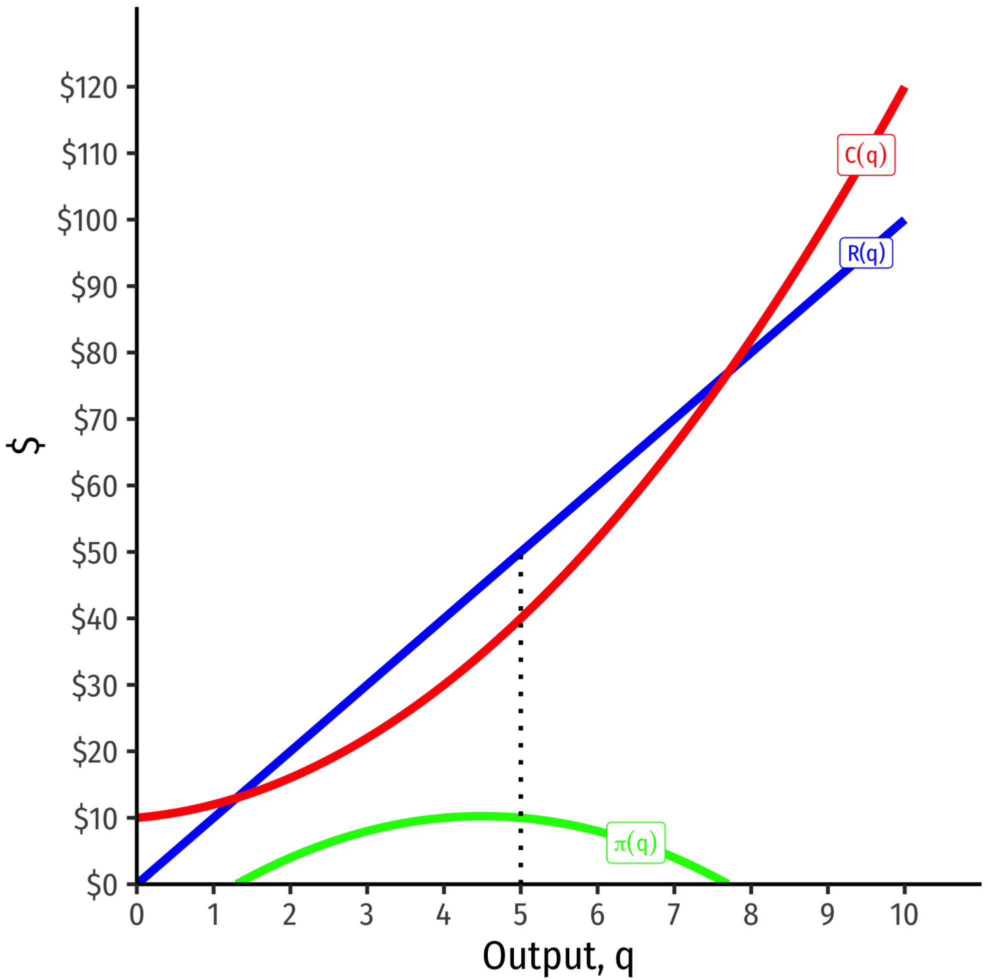


# Visualizing Total Profit As $R(q) - C(q)$

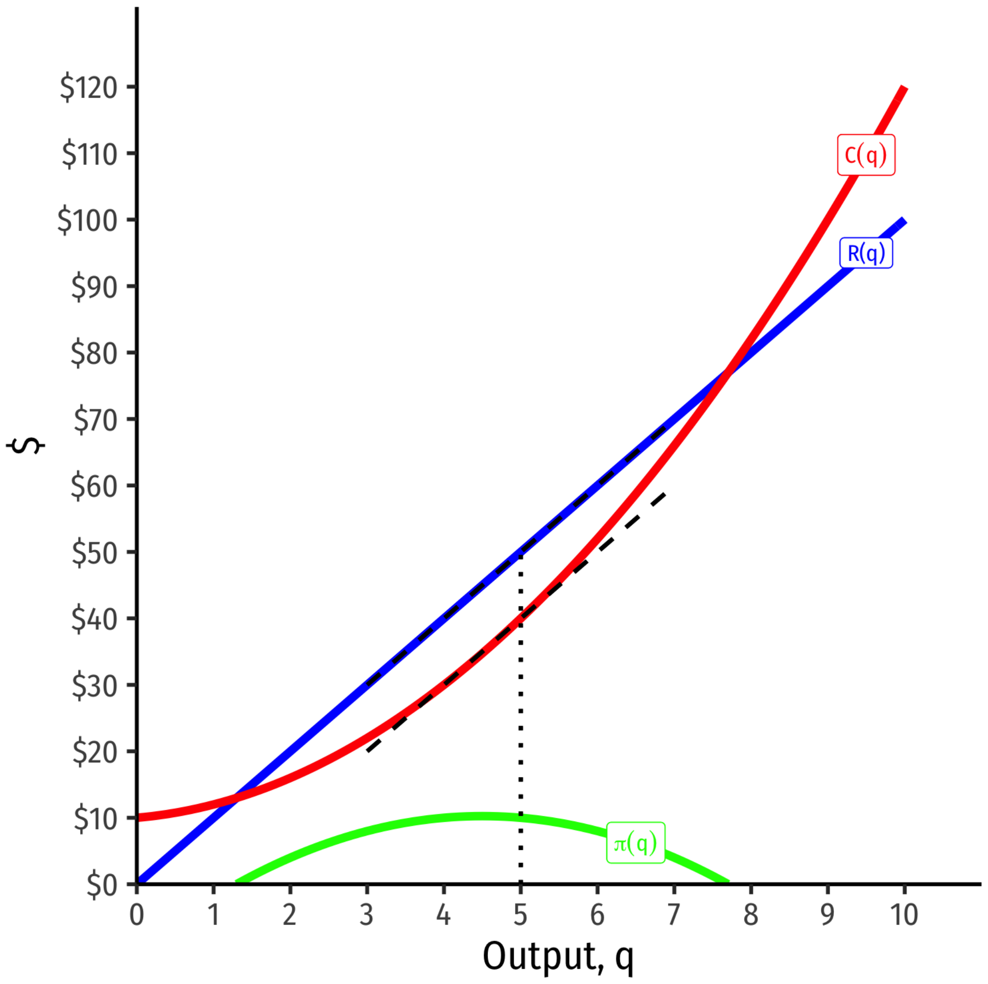
- $\pi(q) = R(q) - C(q)$
- Graph: find $q^*$ to max $\pi \implies q^*$ where max distance between $R(q)$ and $C(q)$



# Visualizing Total Profit As $R(q) - C(q)$

- $\pi(q) = R(q) - C(q)$
- Graph: find $q^*$ to max $\pi \implies q^*$ where max distance between $R(q)$ and $C(q)$
- Slopes must be equal:

$$MR(q) = MC(q)$$

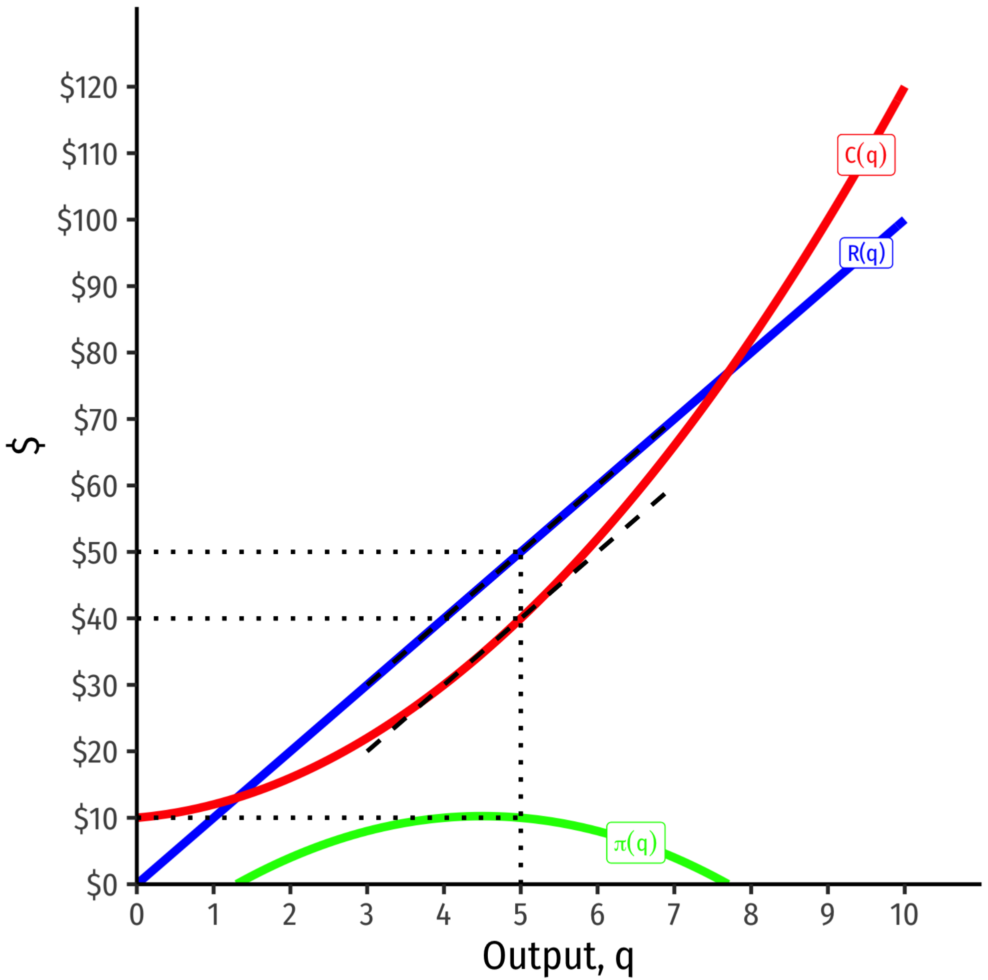


# Visualizing Total Profit As $R(q) - C(q)$

- $\pi(q) = R(q) - C(q)$
- Graph: find $q^*$ to max $\pi \implies q^*$ where max distance between $R(q)$ and $C(q)$
- Slopes must be equal:

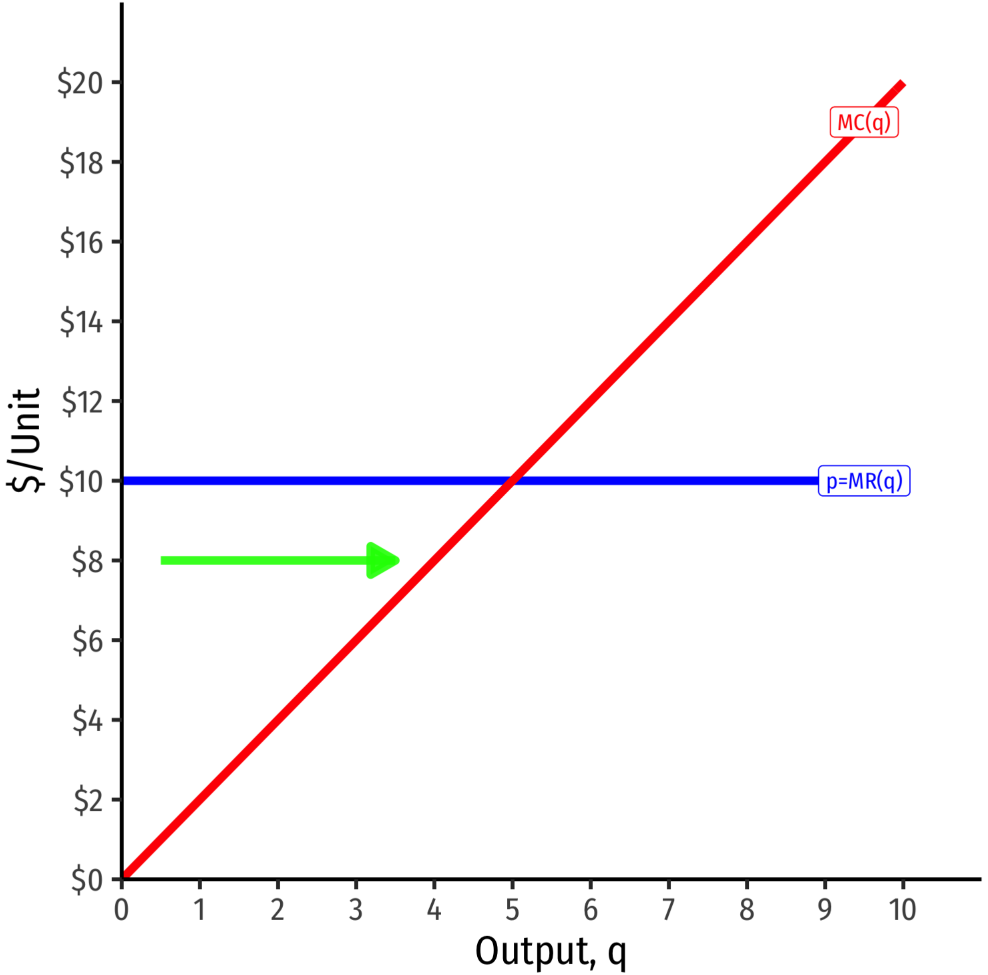
$$MR(q) = MC(q)$$

- At $q^* = 5$:
  - $R(q) = 50$
  - $C(q) = 40$
  - $\pi(q) = 10$



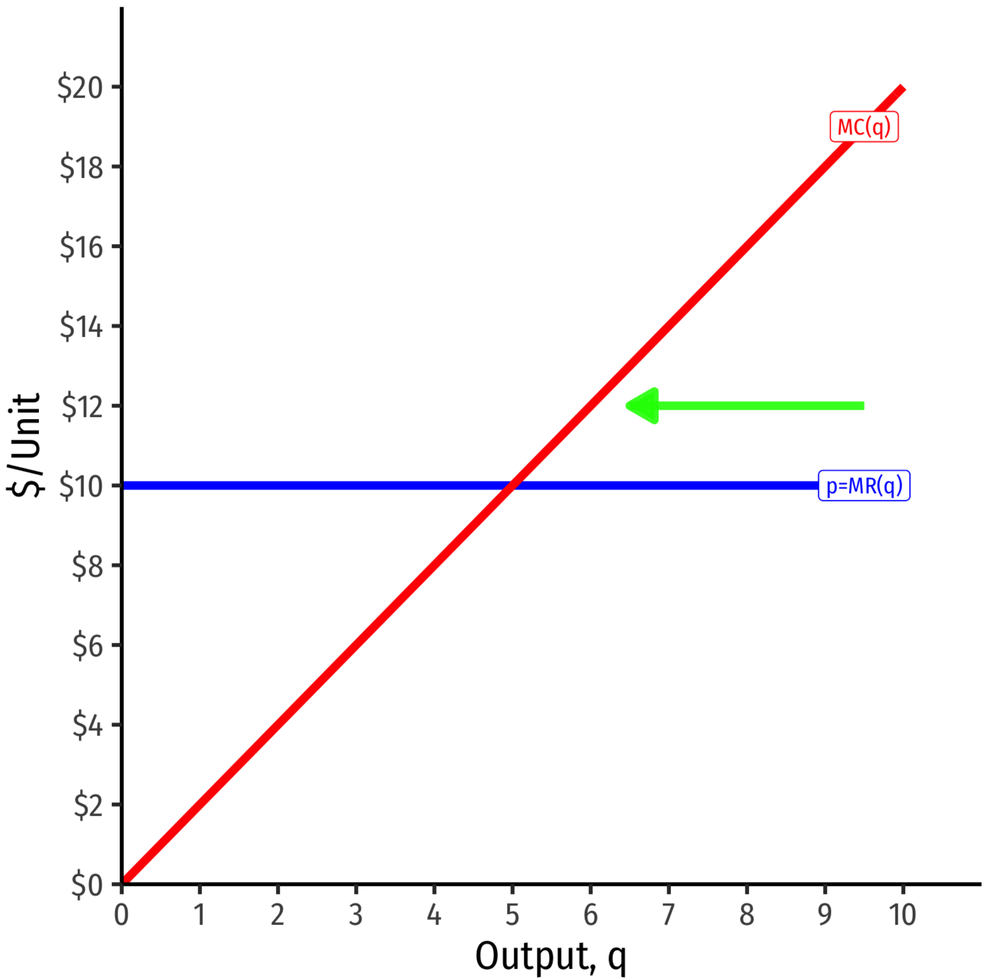
# Visualizing Profit Per Unit As $MR(q)$ and $MC(q)$

- At low output $q < q^*$, can increase $\pi$ by producing *more*: $MR(q) > MC(q)$



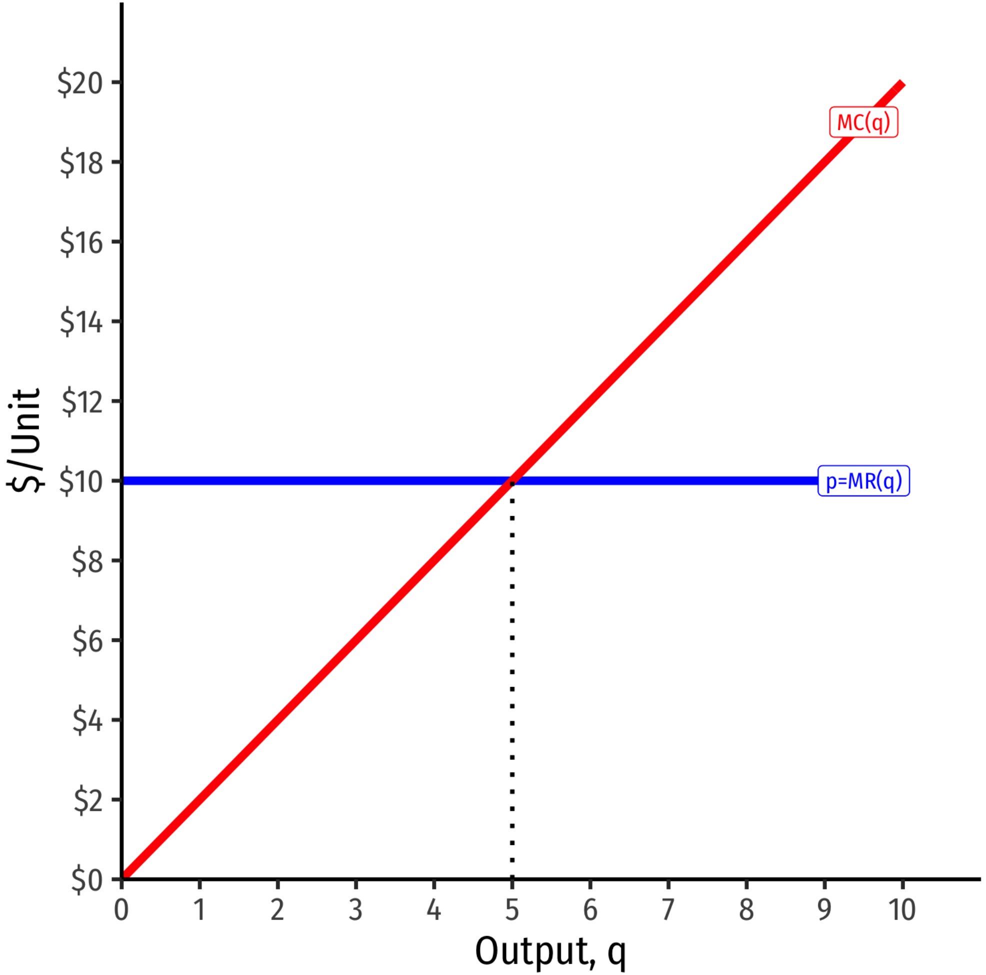
# Visualizing Profit Per Unit As $MR(q)$ and $MC(q)$

- At high output $q > q^*$, can increase $\pi$ by producing *less*: $MR(q) < MC(q)$



# Visualizing Profit Per Unit As $MR(q)$ and $MC(q)$

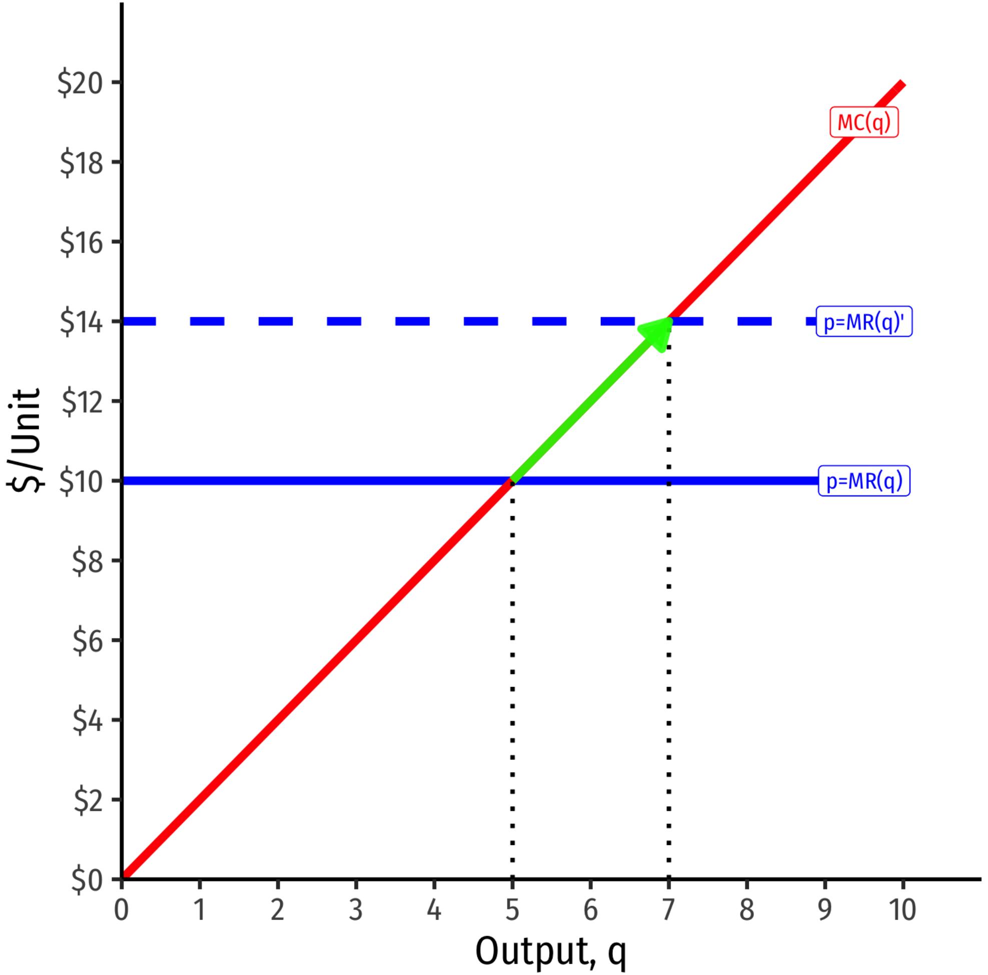
- $\pi$ is *maximized* where $MR(q) = MC(q)$



# Comparative Statics

# If Market Price Changes I

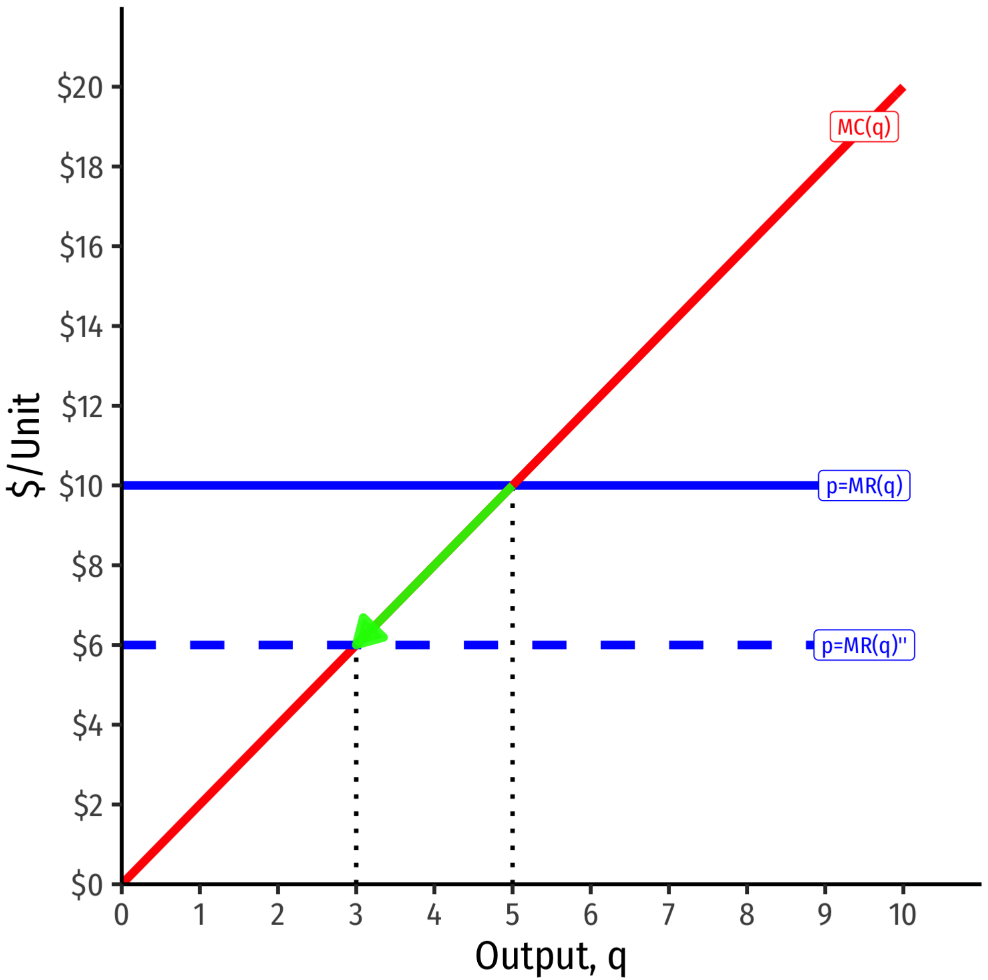
- Suppose the market price **increases**
- Firm (always setting MR=MC) will respond by **producing more**



# If Market Price Changes II

- Suppose the market price **decreases**

- Firm (always setting MR=MC) will respond by **producing less**

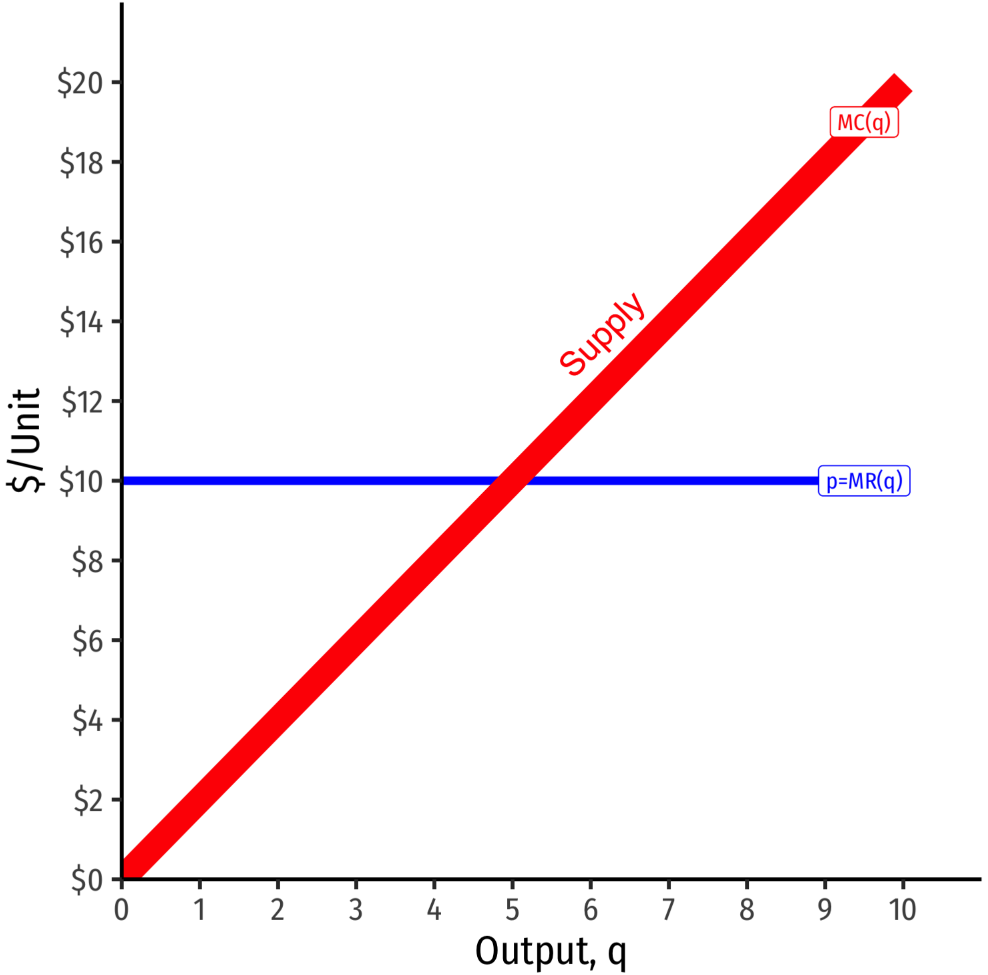


# The Firm's Supply Curve

- **The firm's marginal cost curve is its supply curve**‡

$$p = MC(q)$$

  - How it will supply the optimal amount of output in response to the market price
  - **Firm always sets its price equal to its marginal cost**



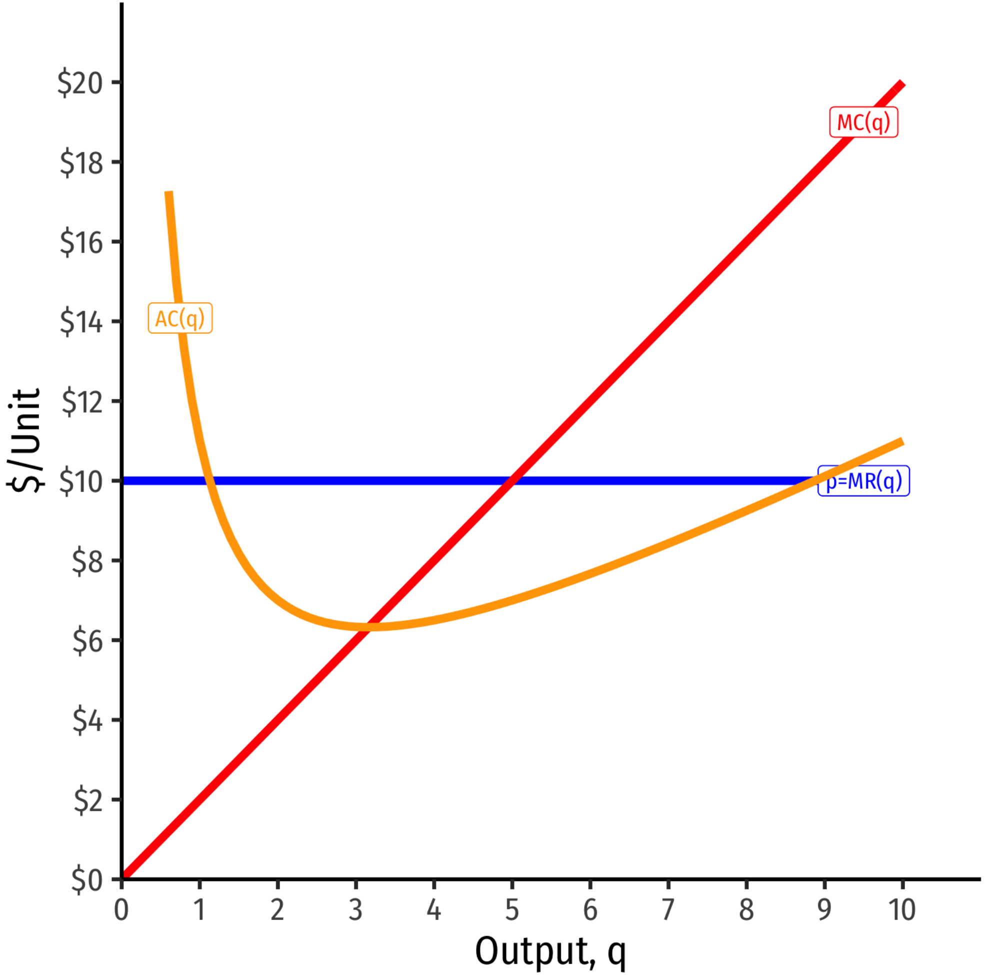

‡ Mostly...there is an important **exception** we will see shortly!

# Calculating Profit

# Calculating (Average) Profit as AR(q)-AC(q)

- Profit is

$$\pi(q) = R(q) - C(q)$$



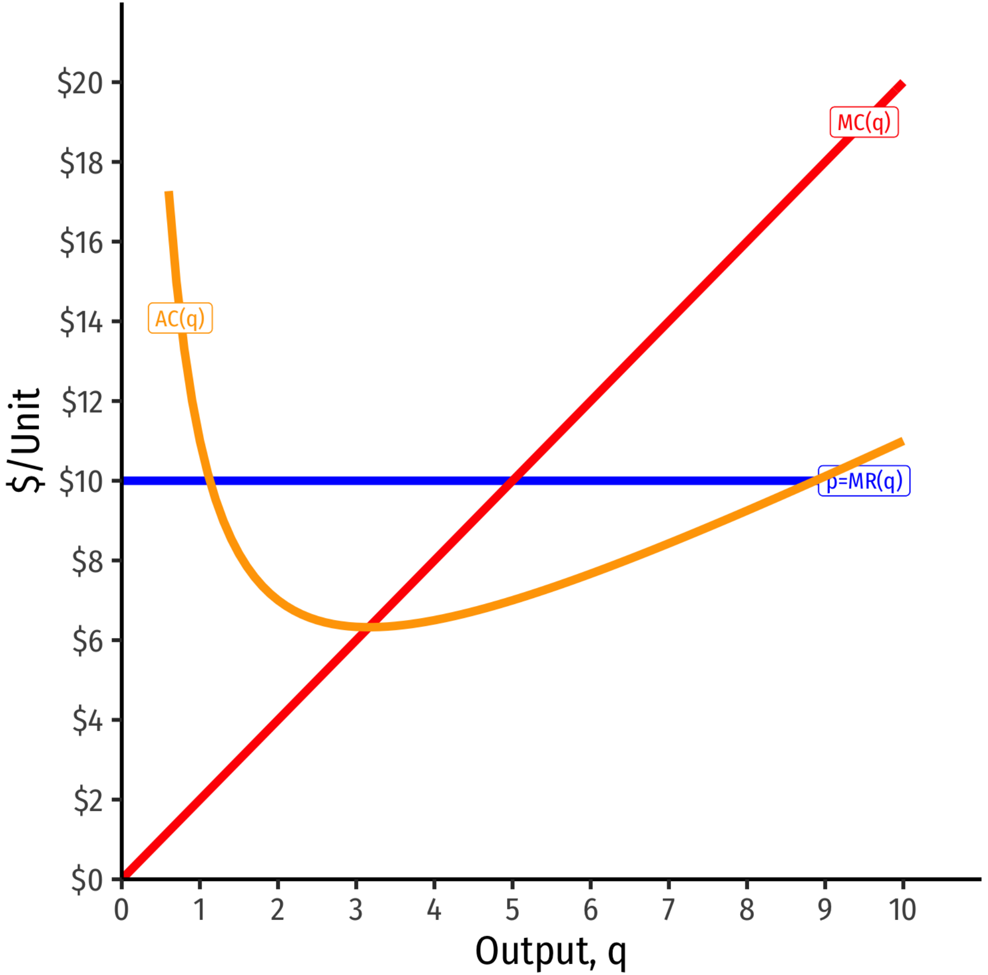
# Calculating (Average) Profit as AR(q)-AC(q)

- Profit is

$$\pi(q) = R(q) - C(q)$$

- Profit per unit can be calculated as:

$$\frac{\pi(q)}{q} = \color{blue}{AR(q)} - \color{orange}{AC(q)}$$
$$= \color{blue}{p} - \color{orange}{AC(q)}$$



# Calculating (Average) Profit as AR(q)-AC(q)

- Profit is

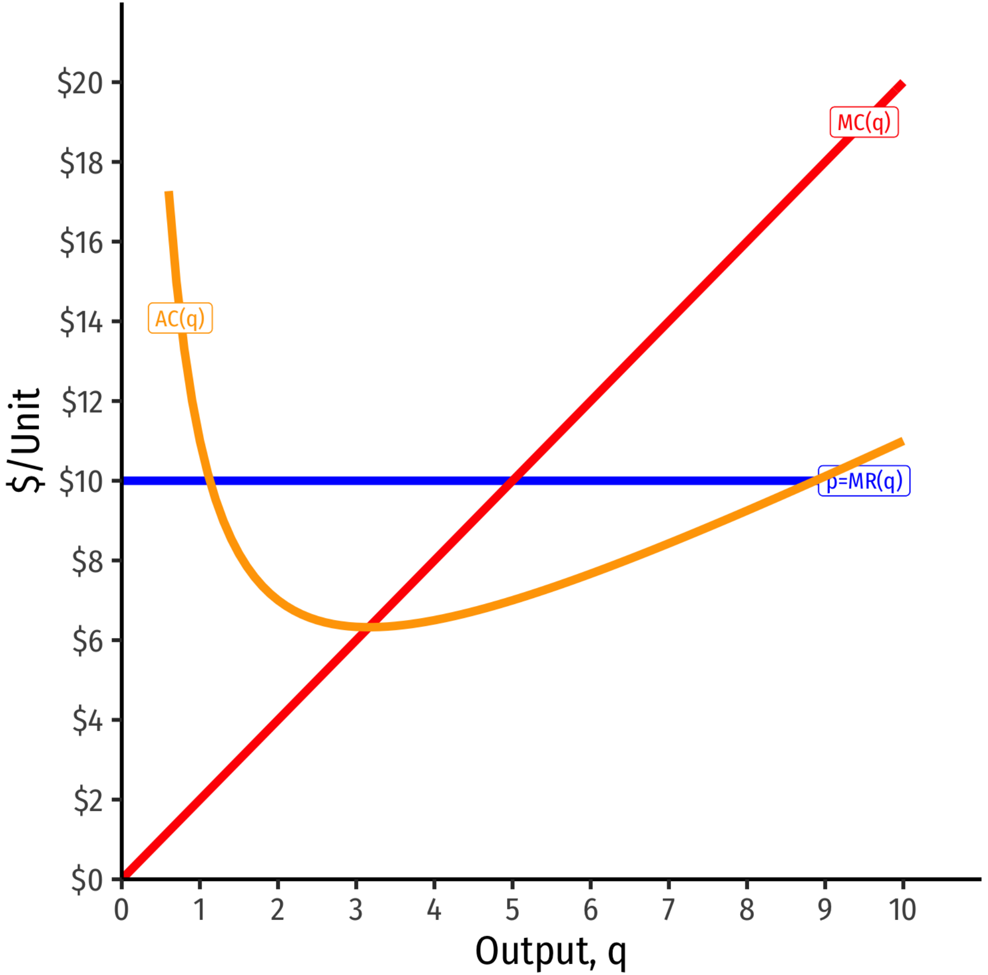
$$\pi(q) = R(q) - C(q)$$

- Profit per unit can be calculated as:

$$\begin{aligned}\frac{\pi(q)}{q} &= AR(q) - AC(q)\\ &= p - AC(q)\end{aligned}$$

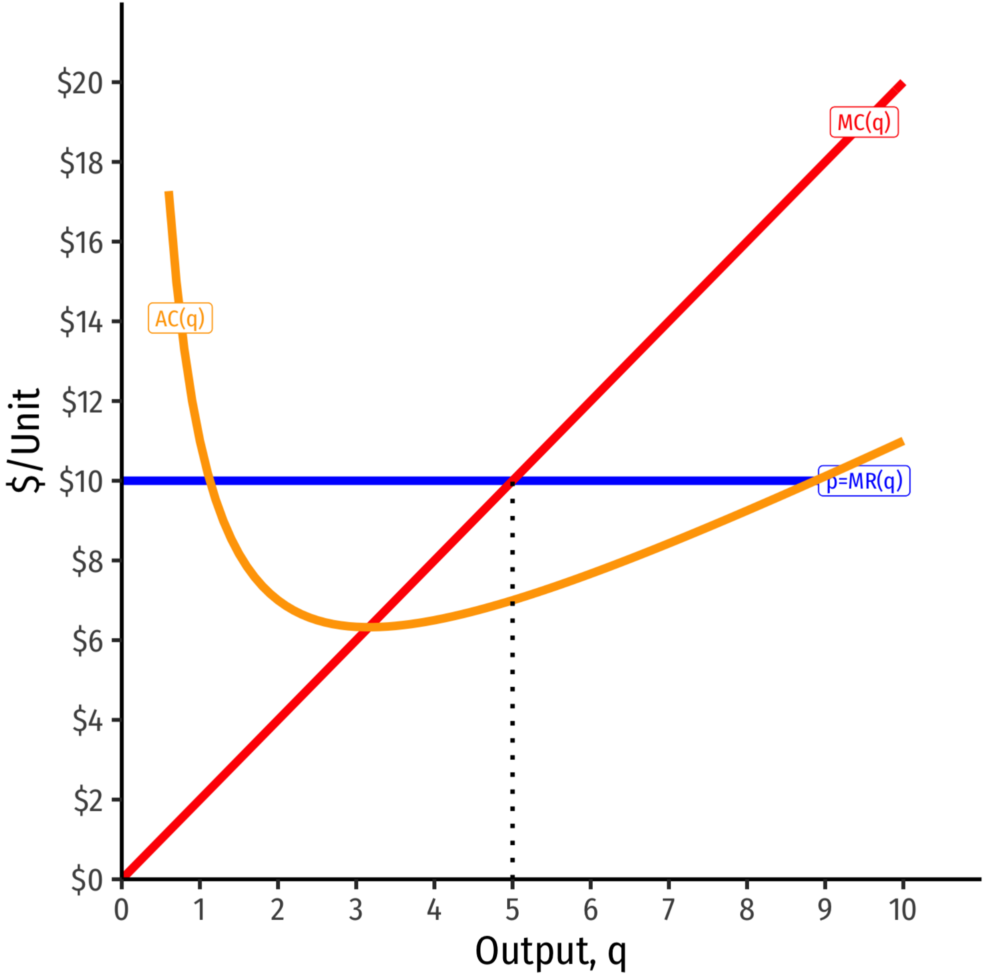
- Multiply by $q$ to get total profit:

$$\pi(q) = q\left[p - AC(q)\right]$$



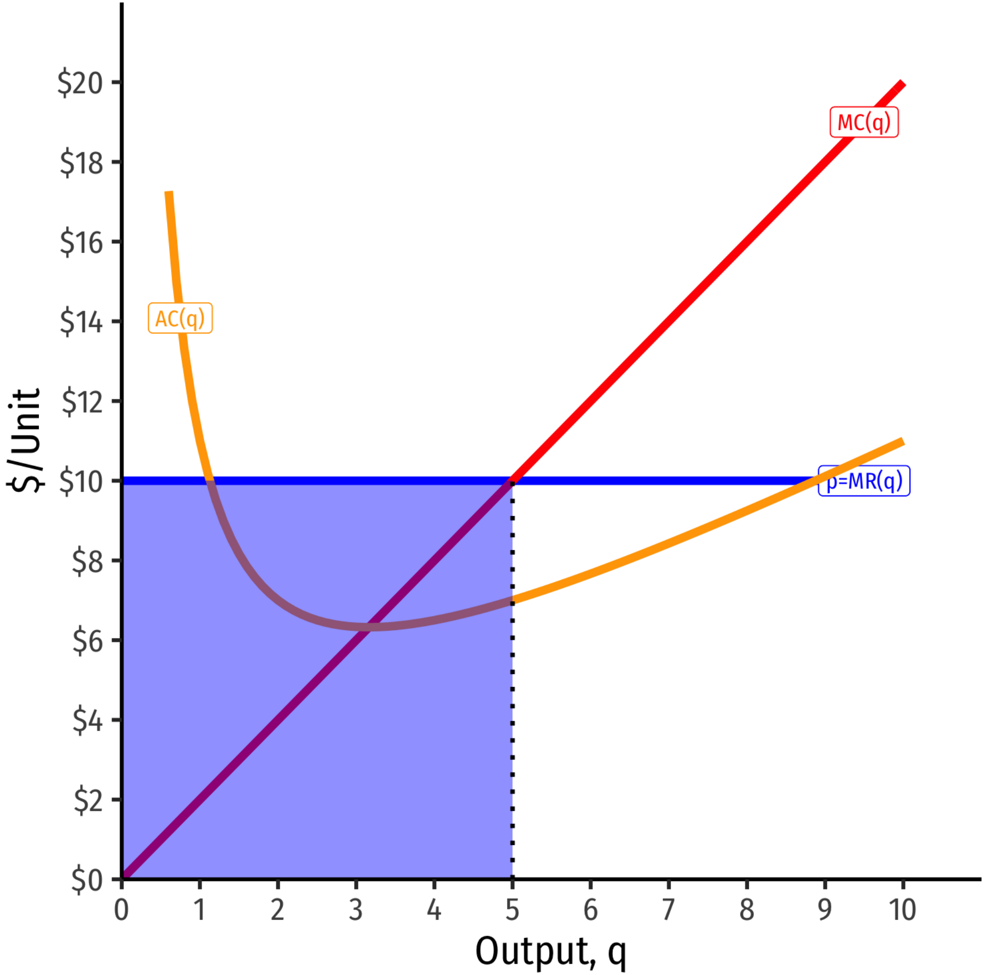
# Calculating (Average) Profit as AR(q)-AC(q)

- At market price of p* = $10
- At q* = 5 (per unit):
- At q* = 5 (totals):



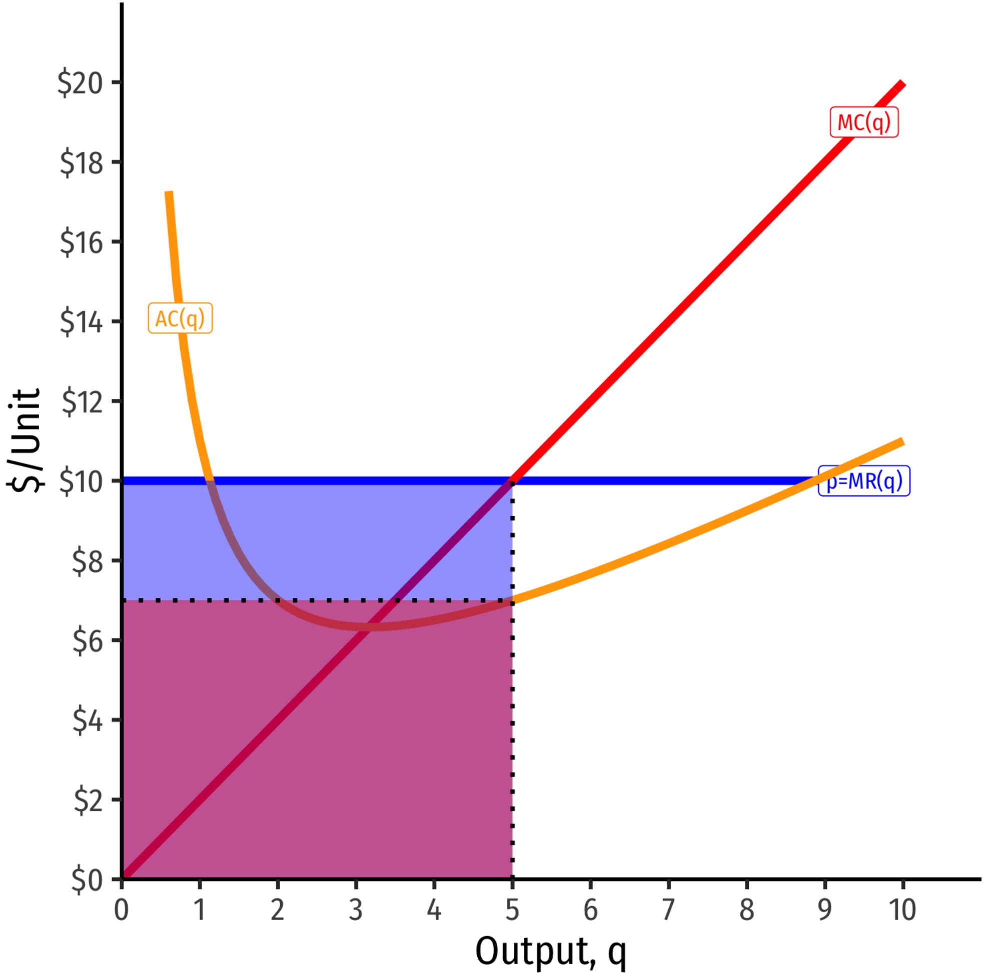
# Calculating (Average) Profit as AR(q)-AC(q)

- At market price of p* = $10
- At q* = 5 (per unit):
  - AR(5) = $10/unit
- At q* = 5 (totals):
  - R(5) = $50



# Calculating (Average) Profit as AR(q)-AC(q)

- At market price of p* = $10
- At q* = 5 (per unit):
  - AR(5) = $10/unit
  - AC(5) = $7/unit
- At q* = 5 (totals):
  - R(5) = $50
  - C(5) = $35

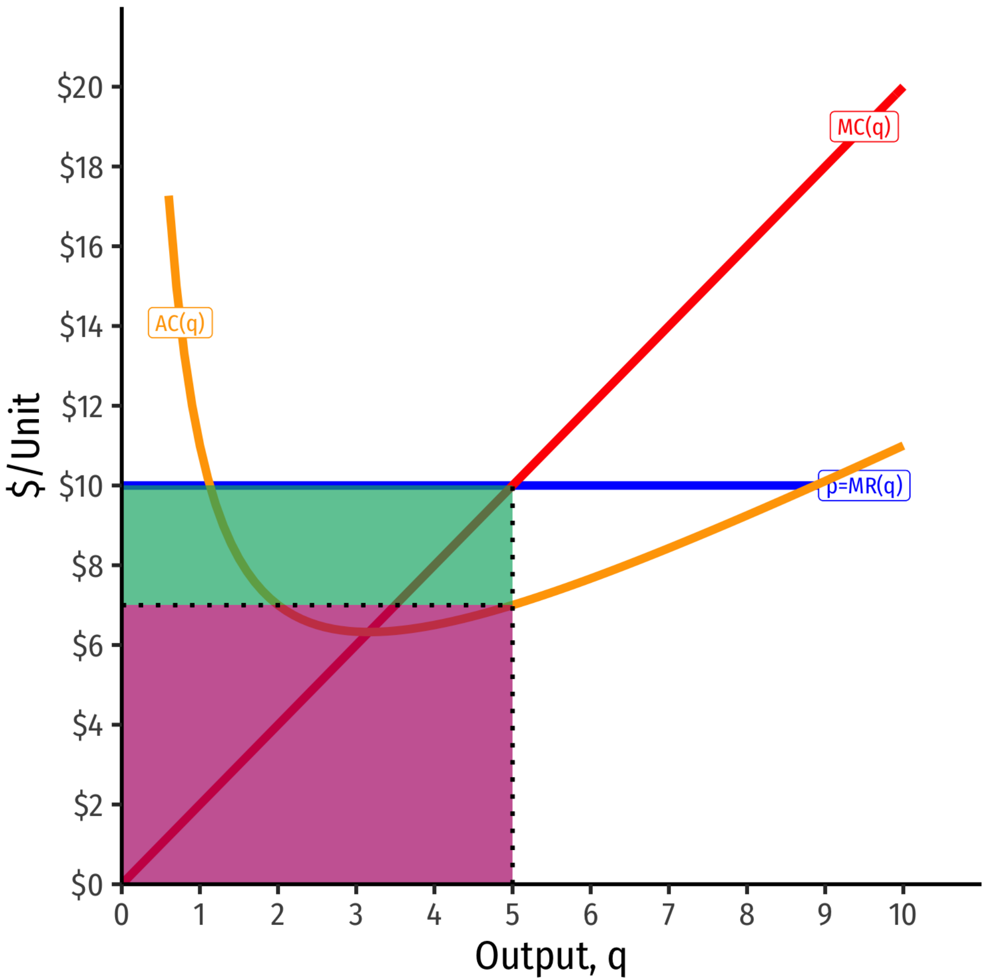


# Calculating (Average) Profit as AR(q)-AC(q)

- At market price of $p^* = \$10$

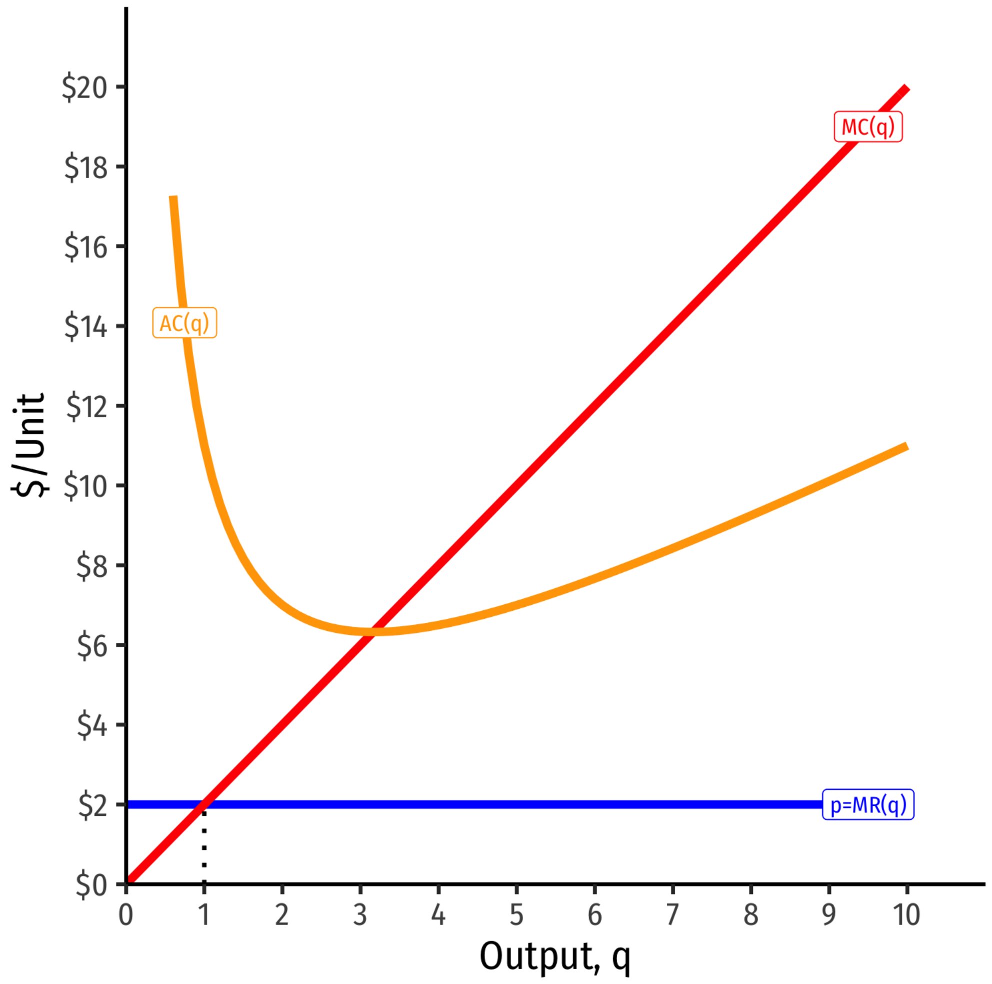
- At $q^* = 5$ (per unit):
  - AR(5) = \$10/unit
  - AC(5) = \$7/unit
  - $A\pi(5)$ = \$3/unit

- At $q^* = 5$ (totals):
  - R(5) = \$50
  - C(5) = \$35
  - $\pi$ = \$15



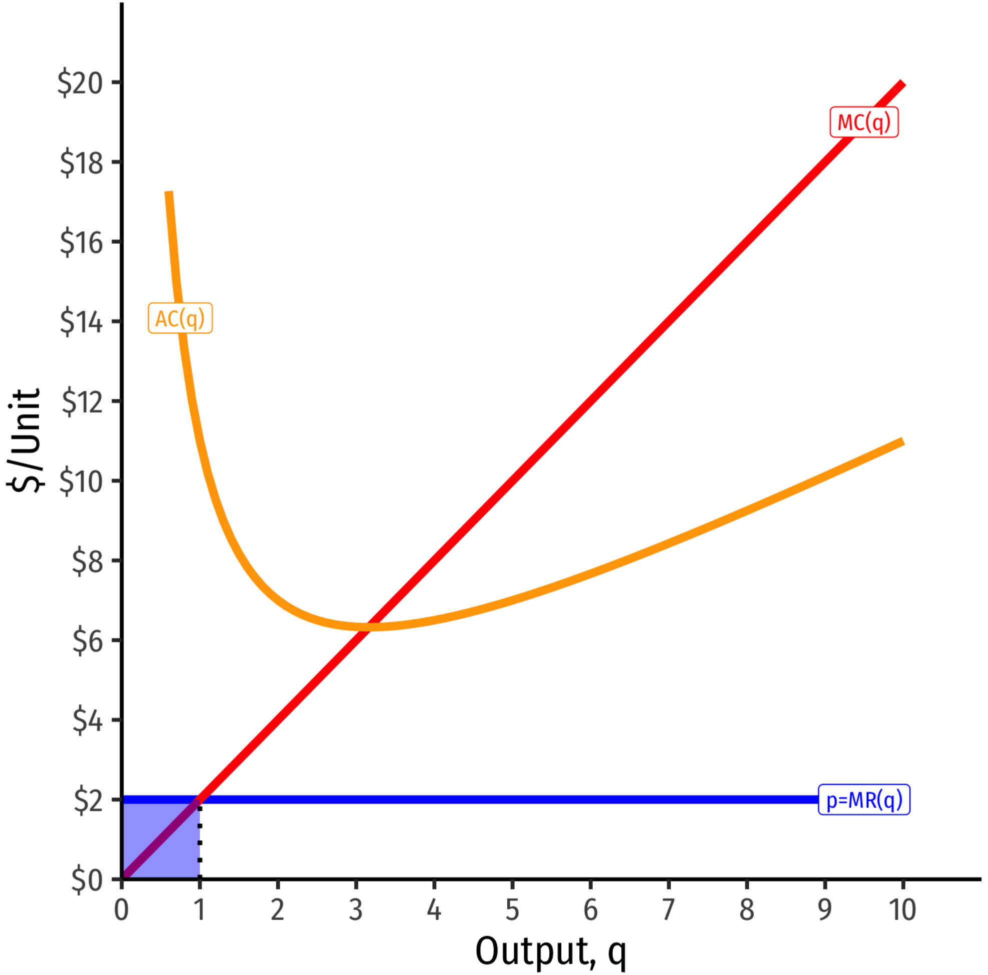
# Calculating (Average) Profit as AR(q)-AC(q)

- At market price of p* = $2
- At q* = 1 (per unit):
- At q* = 1 (totals):



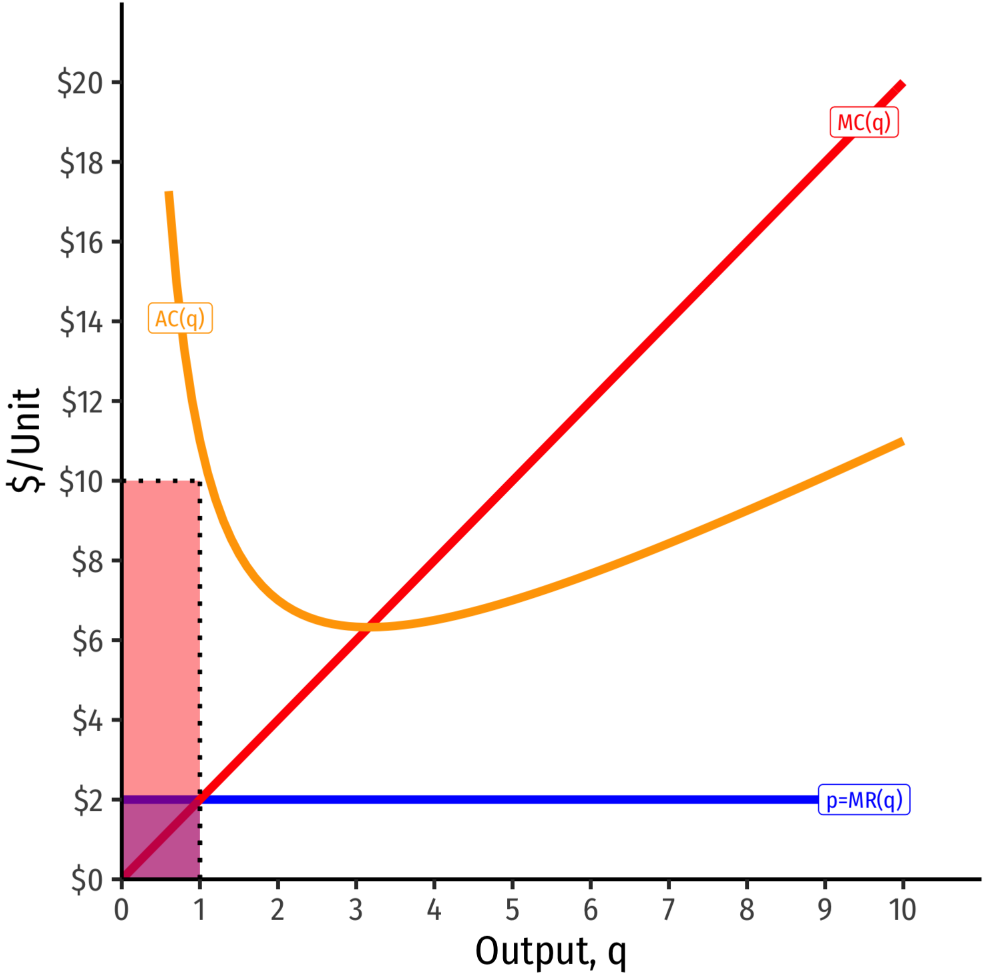
# Calculating (Average) Profit as AR(q)-AC(q)

- At market price of p* = $2
- At q* = 1 (per unit):
  - AR(1) = $2/unit
- At q* = 1 (totals):
  - R(1) = $2



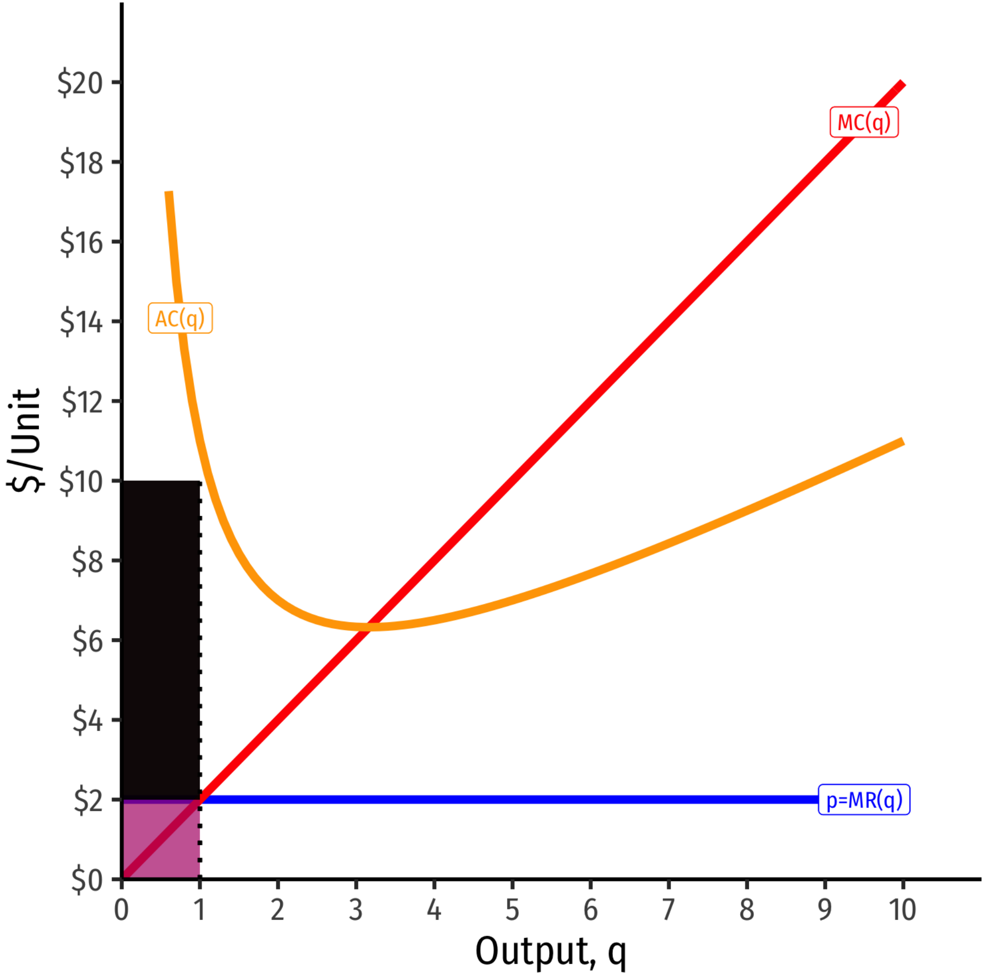
# Calculating (Average) Profit as AR(q)-AC(q)

- At market price of p* = $2
- At q* = 1 (per unit):
  - AR(1) = $2/unit
  - AC(1) = $10/unit
- At q* = 1 (totals):
  - R(1) = $2
  - C(1) = $10



# Calculating (Average) Profit as AR(q)-AC(q)

- At market price of $p^* = \$2$
- At $q^* = 1$ (per unit):
  - $AR(1) = \$2/\text{unit}$
  - $AC(1) = \$10/\text{unit}$
  - $A\pi(1) = -\$8/\text{unit}$
- At $q^* = 1$ (totals):
  - $R(1) = \$2$
  - $C(1) = \$10$
  - $\pi(1) = -\$8$



# Short-Run Shut-Down Decisions

# Short-Run Shut-Down Decisions

- What if a firm's profits at $q^*$ are **negative** (i.e. it earns **losses**)?

- **Should it produce at all?**

# Short-Run Shut-Down Decisions

- Suppose firm chooses to produce **nothing** $(q=0)$:
- If it has **fixed costs** $(f>0)$, its profits are:

$$\pi(q)=pq-C(q)$$

# Short-Run Shut-Down Decisions

- Suppose firm chooses to produce **nothing** $(q = 0)$:
- If it has **fixed costs** $(f > 0)$, its profits are:

$$\pi(q) = pq - C(q)$$
$$\pi(q) = pq - f - VC(q)$$

# Short-Run Shut-Down Decisions

- Suppose firm chooses to produce **nothing** $(q = 0)$:

- If it has **fixed costs** $(f > 0)$, its profits are:

$$\pi(q) = pq - C(q)$$
$$\pi(q) = pq - f - VC(q)$$
$$\pi(0) = -f$$

i.e. it (still) pays its fixed costs

# Short-Run Shut-Down Decisions

- A firm should choose to produce **no output** $(q = 0)$ only when:

$$\pi \text{ from producing} < \pi \text{ from not producing}$$

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$$\begin{aligned}\pi \text{ from producing} &< \pi \text{ from not producing} \\ \pi(q) &< -f\end{aligned}$$

# Short-Run Shut-Down Decisions

- A firm should choose to produce **no output** $(q = 0)$ only when:

$$\begin{aligned}\pi \text{ from producing} &< \pi \text{ from not producing} \\ \pi(q) &< -f \\ pq - VC(q) - f &< -f\end{aligned}$$

# Short-Run Shut-Down Decisions

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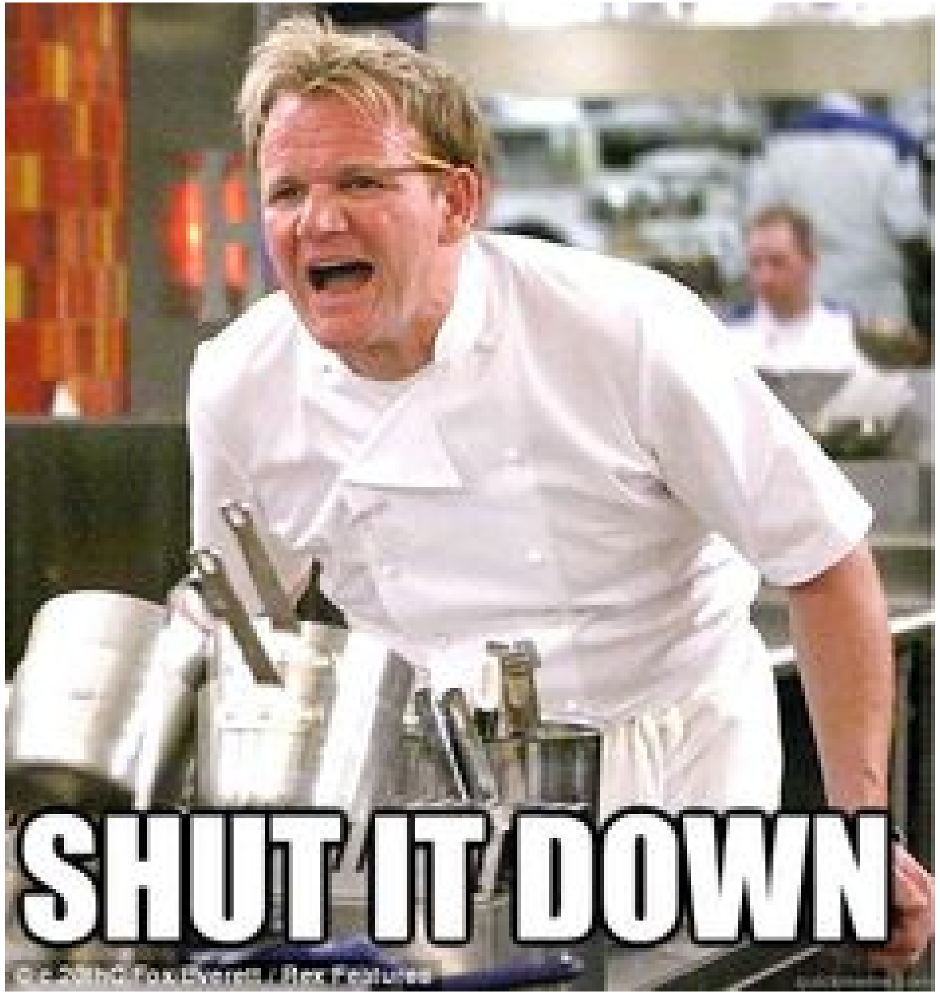
$$\begin{aligned}
\pi \text{ from producing} &< \pi \text{ from not producing} \\
\pi(q) &< -f \\
pq - VC(q) - f &< -f \\
pq - VC(q) &< 0 \\
pq &< VC(q)
\end{aligned}$$

# Short-Run Shut-Down Decisions

- A firm should choose to produce **no output** $(q=0)$ only when:

$$\begin{aligned}\pi \text{ from producing} &< \pi \text{ from not producing}\\ \pi(q) &< -f\\ pq - VC(q) - f &< -f\\ pq - VC(q) &< 0\\ pq &< VC(q)\\ p &< AVC(q)\end{aligned}$$

- **Shut down price**: firm will shut down production *in the short run* when $p < AVC(q)$



# The Firm’s Short Run Supply Decision

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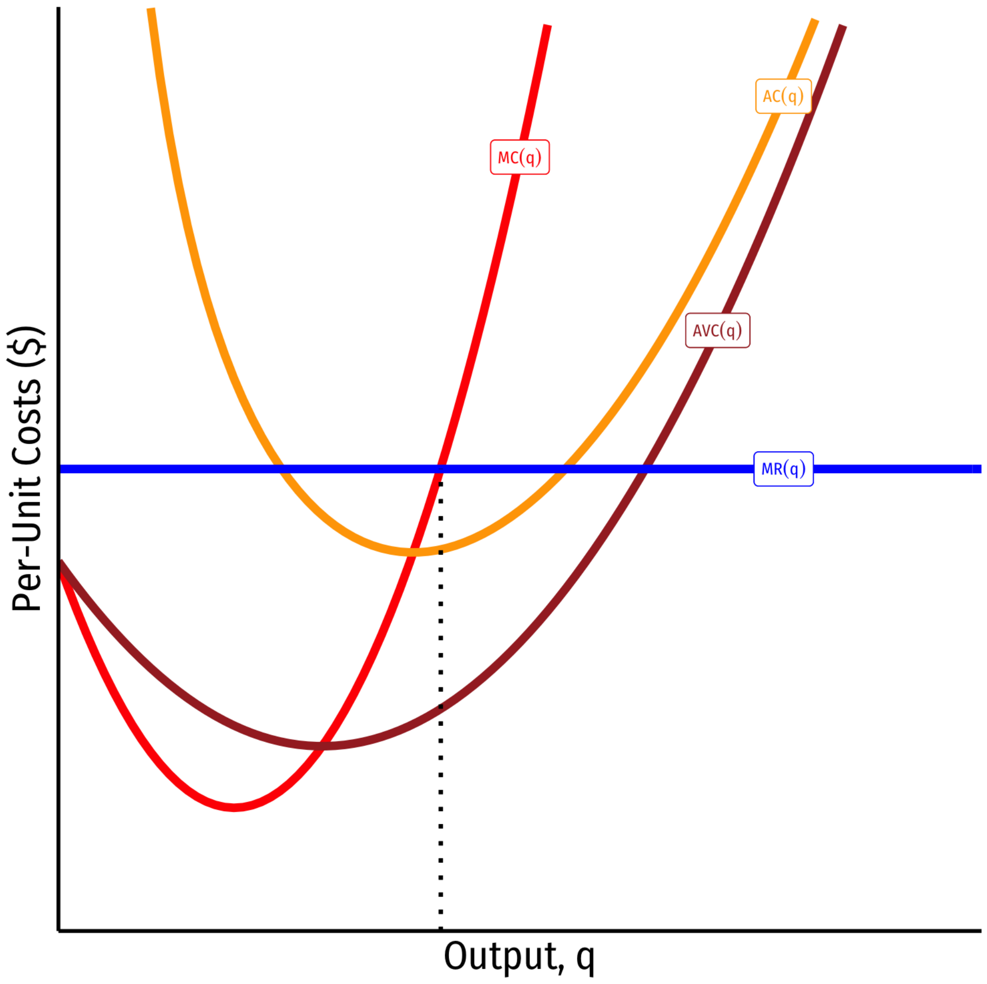

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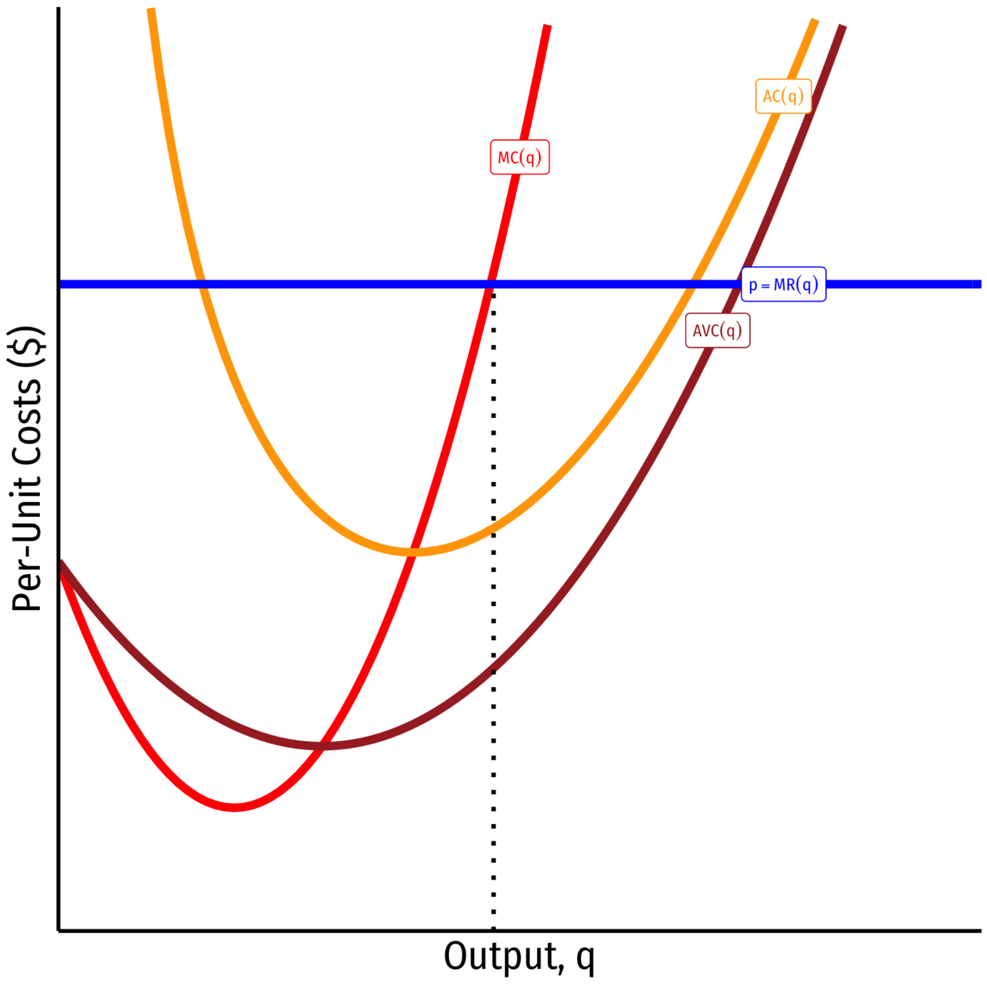

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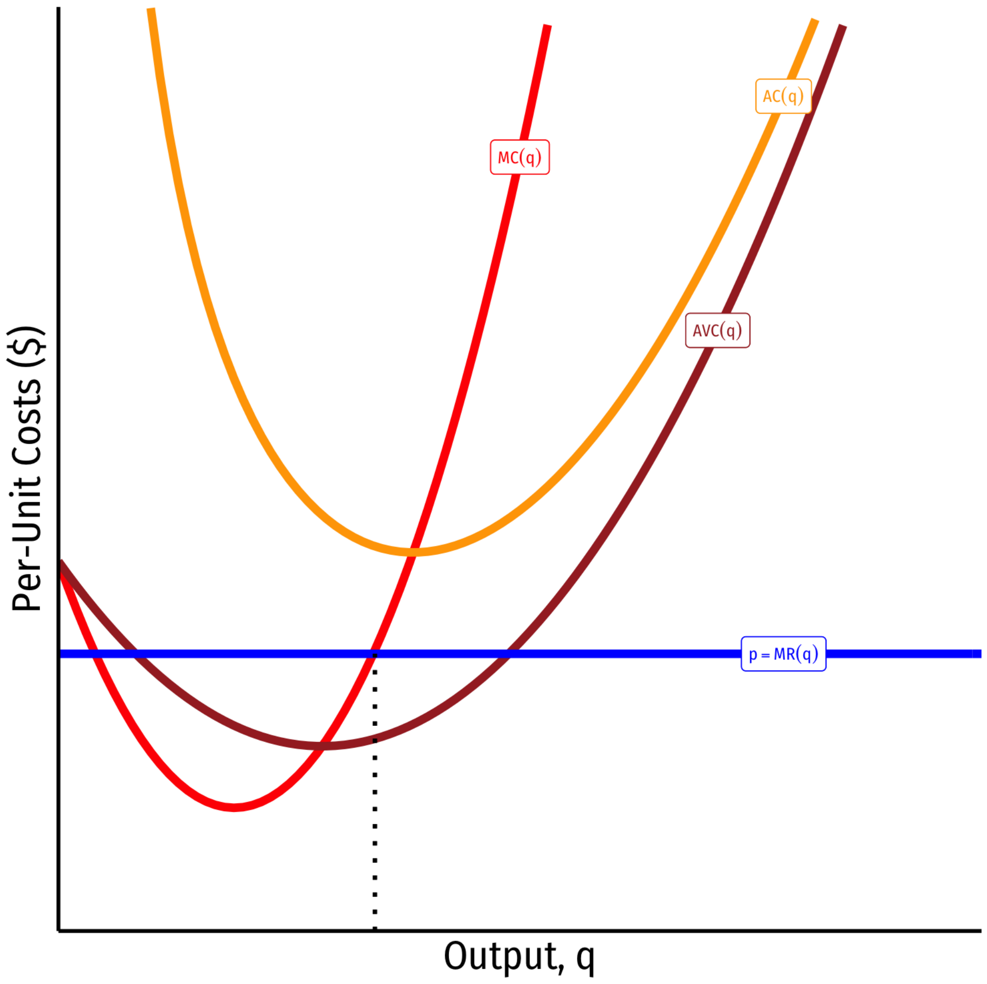

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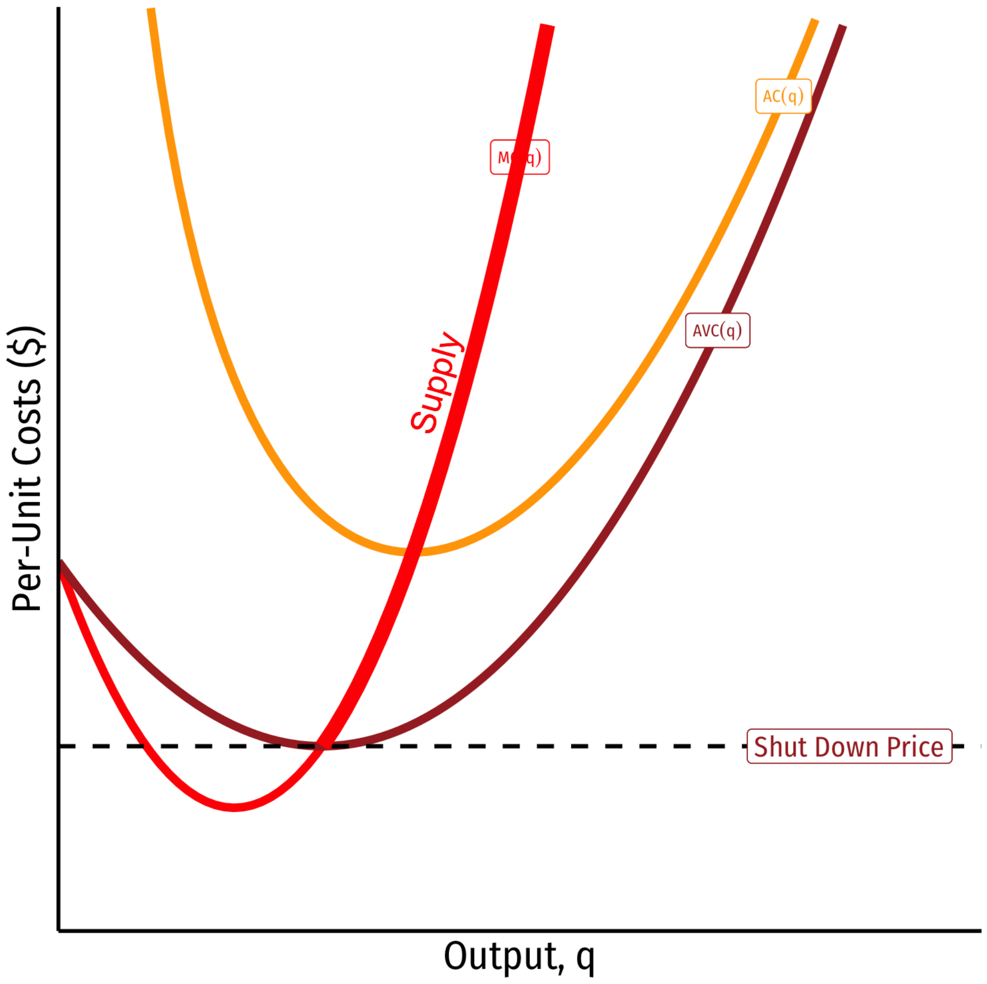

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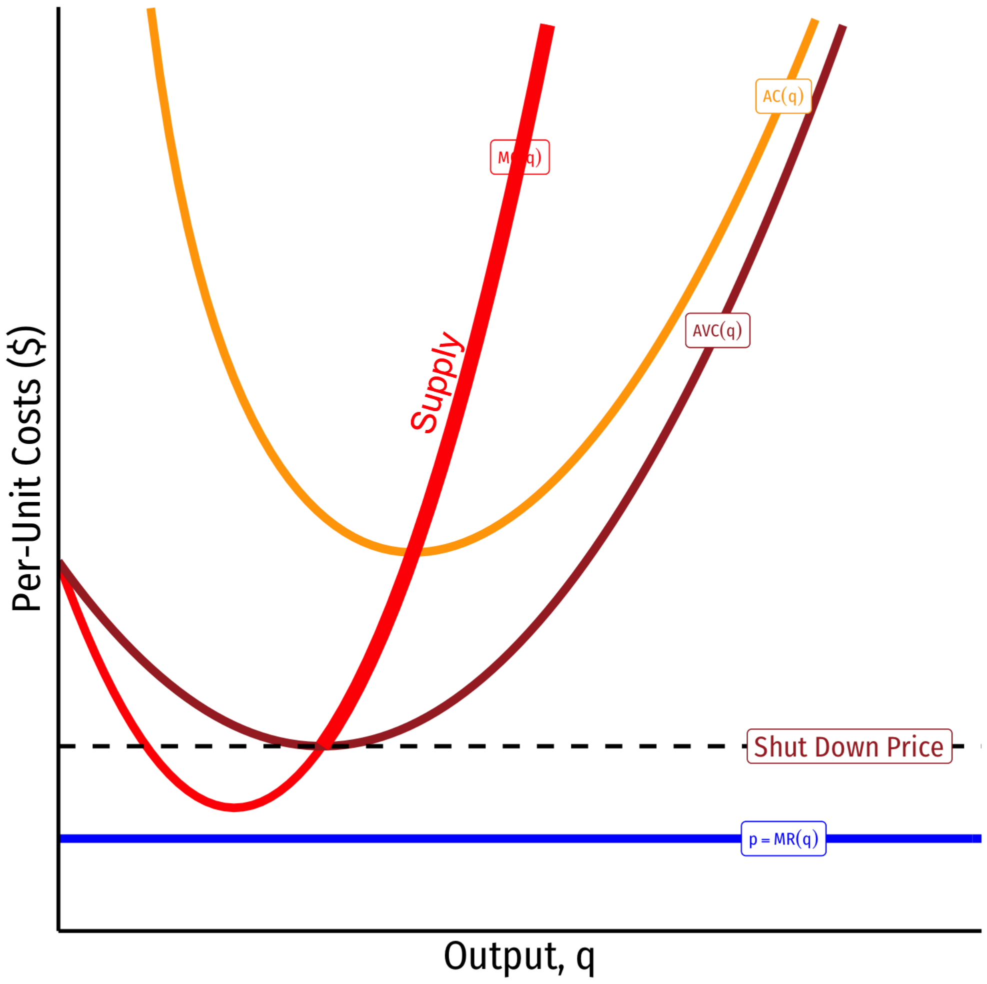

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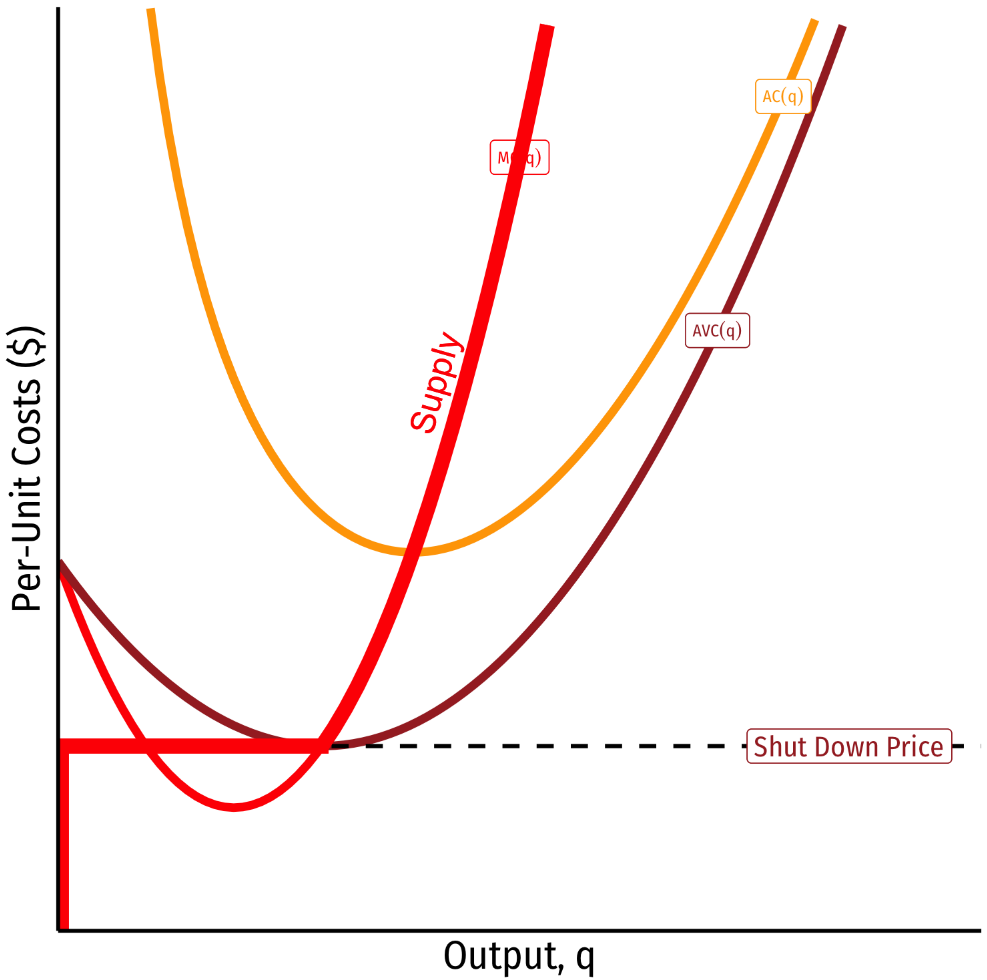
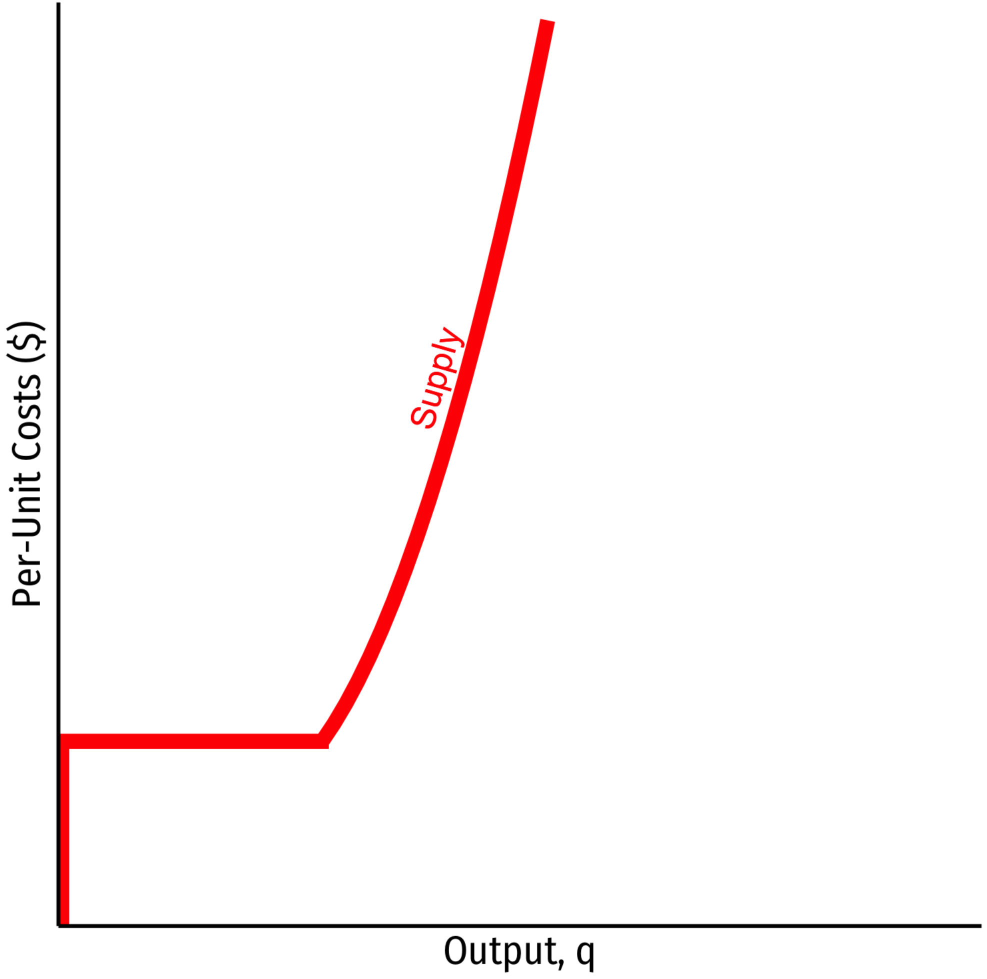



Firm's short run supply curve:

$$\begin{cases} p = MC(q) & \text{if } p \geq AVC \\ q = 0 & \text{If } p < AVC \end{cases}$$

# The Firm's Short Run Supply Decision




Firm's short run supply curve:

$$\begin{cases} p = MC(q) & \text{if } p \geq AVC \\ q = 0 & \text{If } p < AVC \end{cases}$$

# Summary:

**1. Choose $q^*$ such that $MR(q) = MC(q)$**

**2. Profit $\pi = q[p - AC(q)]$**

**3. Shut down if $p < AVC(q)$**

Firm's short run (inverse) supply:

$$\begin{cases} p = MC(q) & \text{if } p \geq AVC \\ q = 0 & \text{If } p < AVC \end{cases}$$

# Choosing the Profit-Maximizing Output $q^*$: Example

**Example**: Bob's barbershop gives haircuts in a very competitive market, where barbers cannot differentiate their haircuts. The current market price of a haircut is $15. Bob's daily short run costs are given by:

$$C(q) = 0.5q^2$$
$$MC(q) = q$$

1. How many haircuts per day would maximize Bob's profits?

2. How much profit will Bob earn per day?

3. Find Bob's shut down price.