2.2 — Short Run and Long Run

ECON 306 • Microeconomic Analysis • Fall 2021

Ryan Safner

Assistant Professor of Economics

safner@hood.edu

ryansafner/microF21

microF21.classes.ryansafner.com

Outline

The "Runs" of Production

- "Time"-frame usefully divided between short vs. long run analysis

- **Short run**: at least one factor of production is **fixed** (too costly to change)

$$q = f(\bar{k}, l)$$

  - Assume **capital** is fixed (i.e. number of factories, storefronts, etc)
  - Short-run decisions only about using **labor**

The “Runs” of Production

- “Time”-frame usefully divided between short vs. long run analysis

- **Long run**: all factors of production are **variable** (can be changed)

$$q = f(k, l)$$

Production in the Short Run

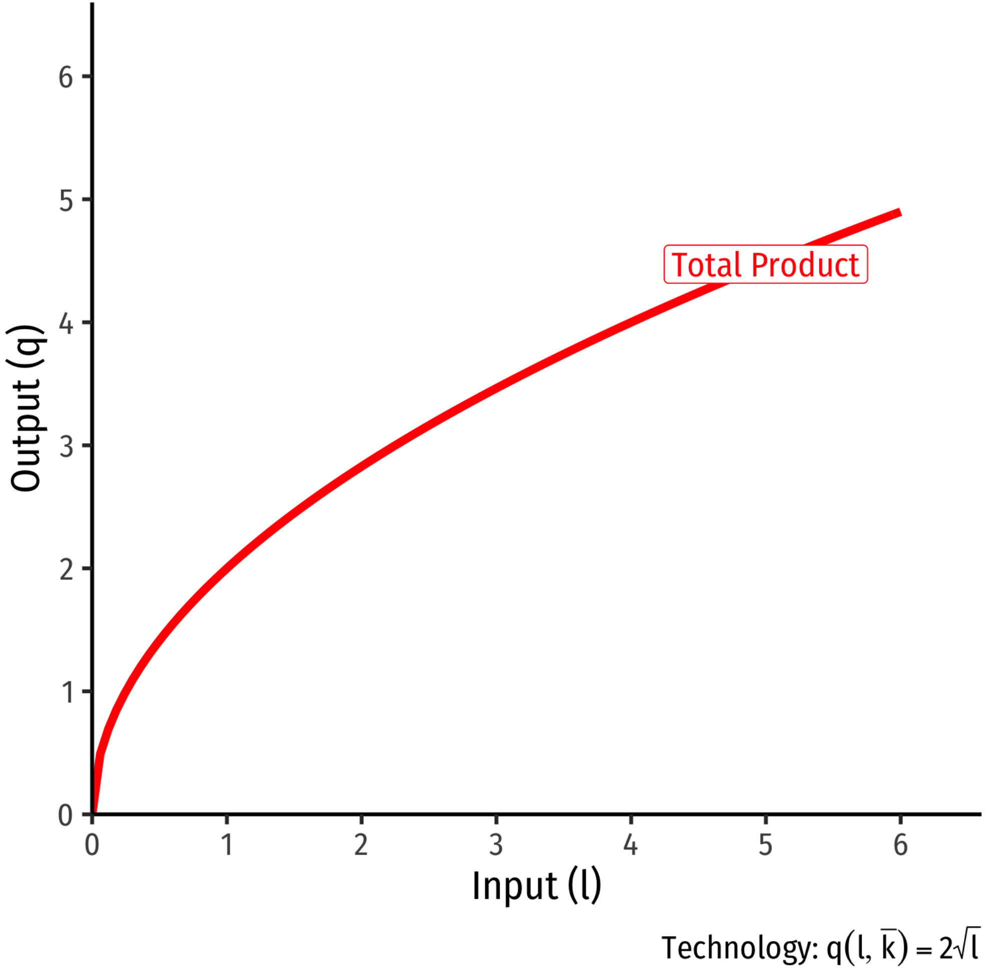
Production in the Short Run: Example

Example: Consider a firm with the production function

$$q = k^{0.5} l^{0.5}$$

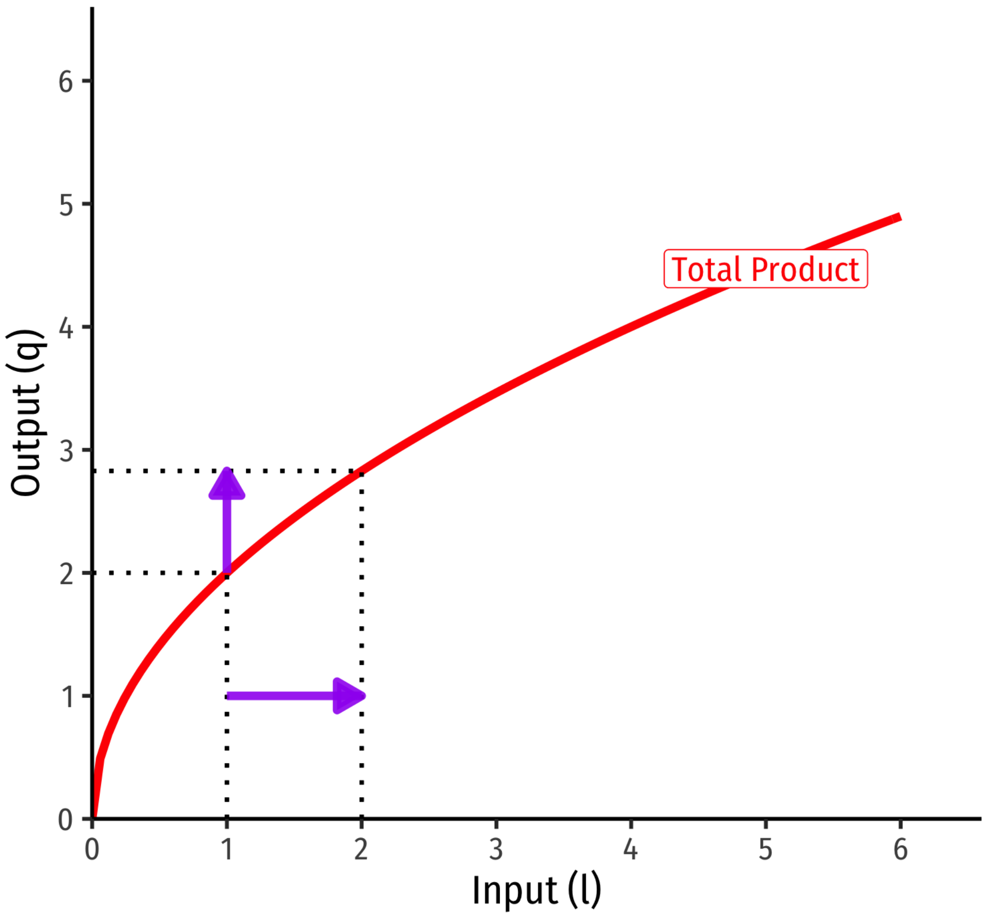
- Suppose in the short run, the firm has 4 units of capital.

1. Derive the short run production function.
2. What is the total product (output) that can be made with 4 workers?
3. What is the total product (output) that can be made with 5 workers?



Marginal Products

- The **marginal product** of an input is the *additional* output produced by *one more unit* of that input (*holding all other inputs constant*)
- Like marginal utility
- Similar to marginal utilities, I will give you the marginal product equations



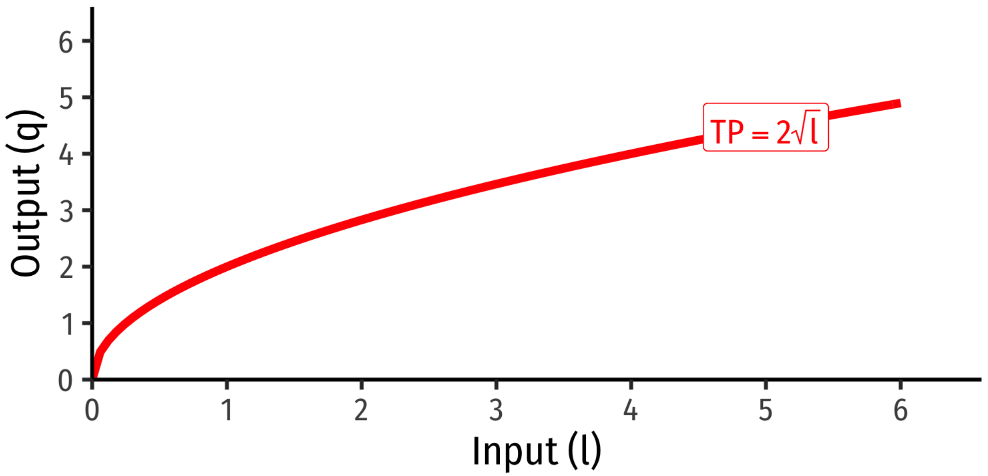
Technology: $q(l, \bar{k}) = 2\sqrt{l}$

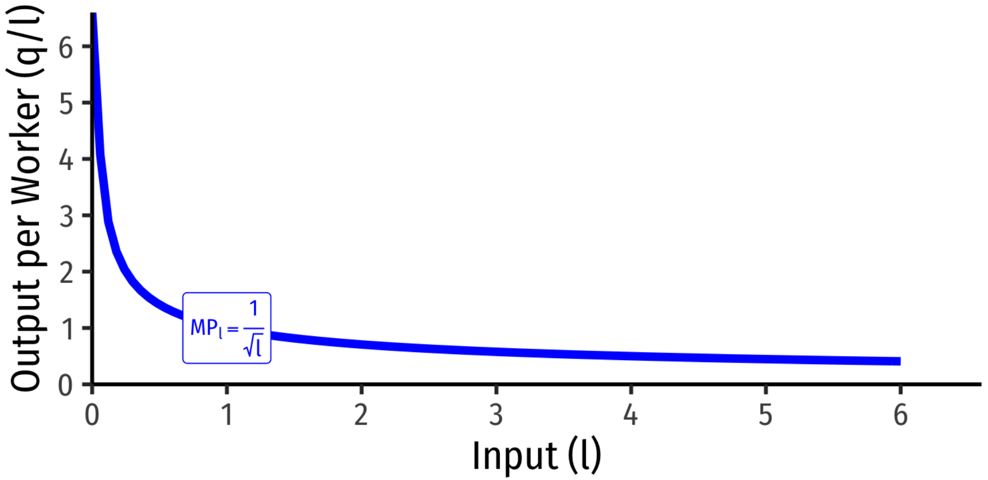
Marginal Product of Labor

- **Marginal product of labor (MP_l)**: additional output produced by adding one more unit of labor (holding k constant)

$$MP_l = \frac{\Delta q}{\Delta l}$$

- MP_l is slope of TP at each value of l!
    - Note: via calculus: $\frac{\partial q}{\partial l}$



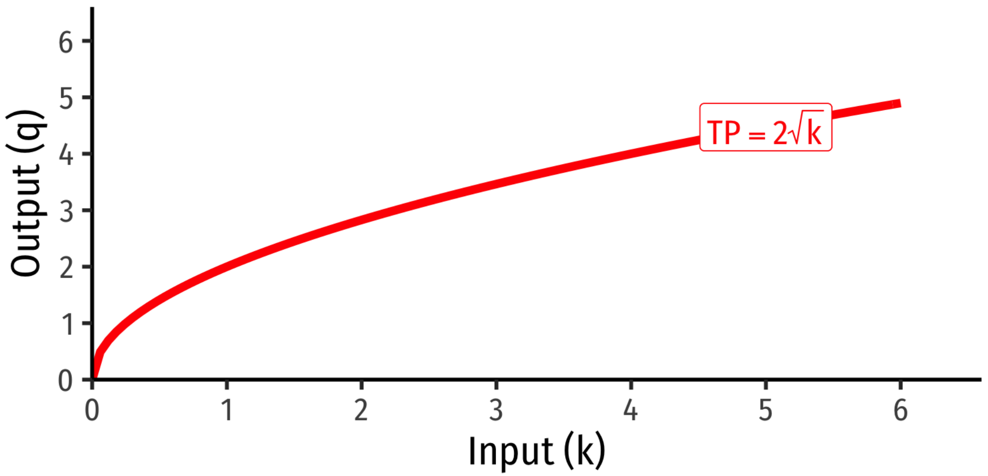


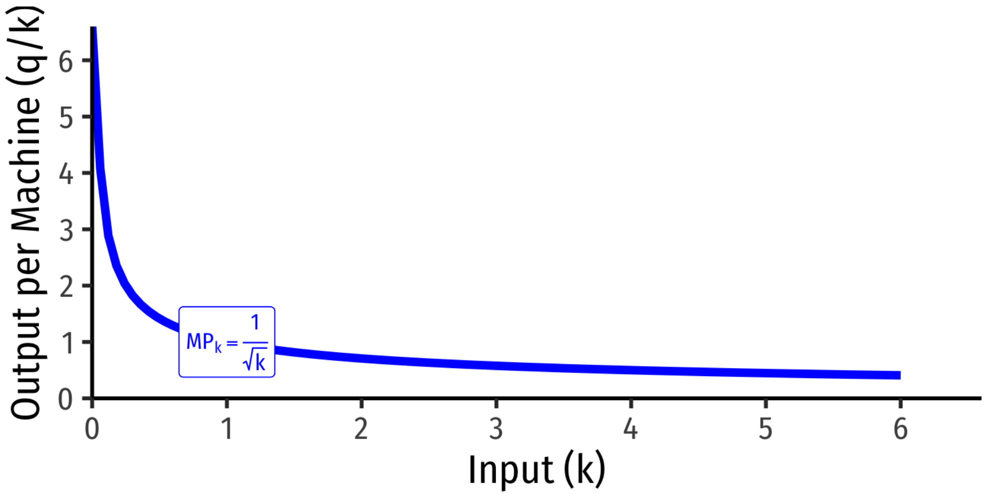
Marginal Product of Capital

- **Marginal product of capital (MP_k)**: additional output produced by adding one more unit of capital (holding l constant)

$$MP_k = \frac{\Delta q}{\Delta k}$$

- MP_k is slope of TP at each value of k!
  - Note: via calculus: $\frac{\partial q}{\partial k}$
- Note we don't consider capital in the short run!

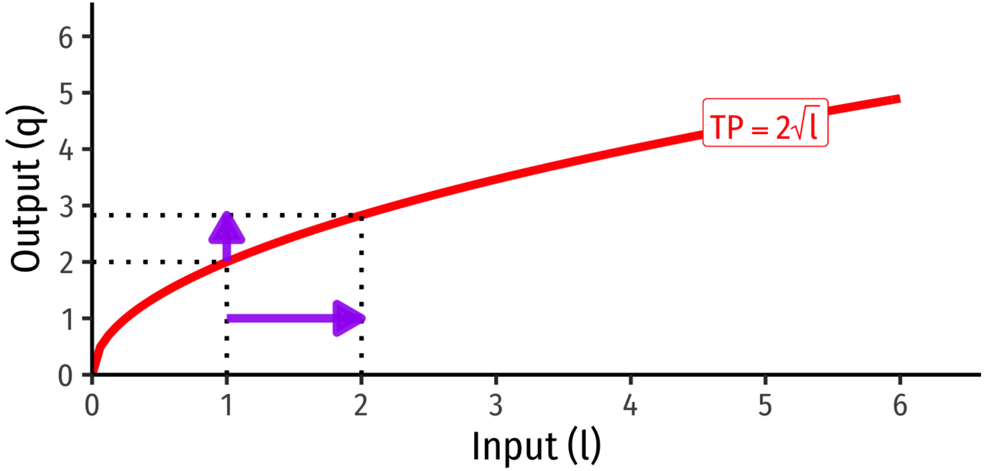


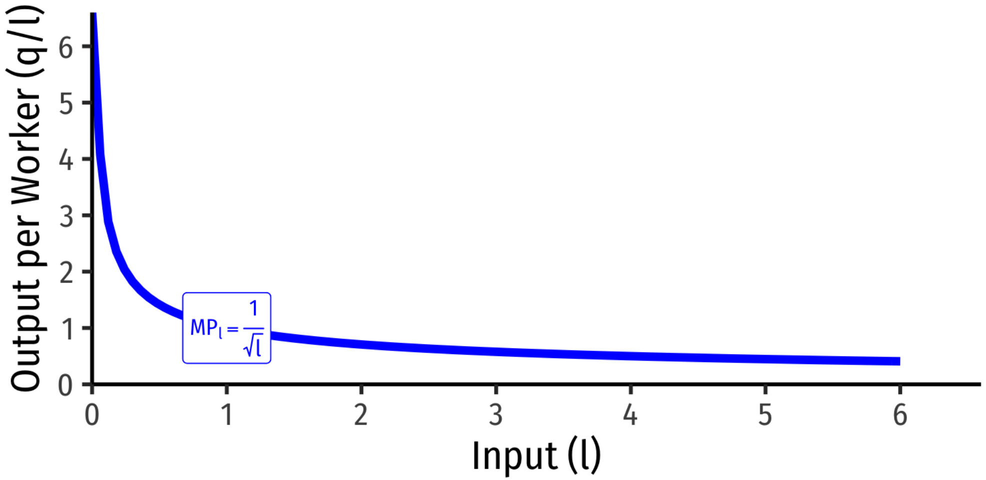


Diminishing Returns

- **Law of Diminishing Returns**: adding more of one factor of production **holding all others constant** will result in successively lower increases in output

- In order to increase output, firm will need to increase *all* factors!

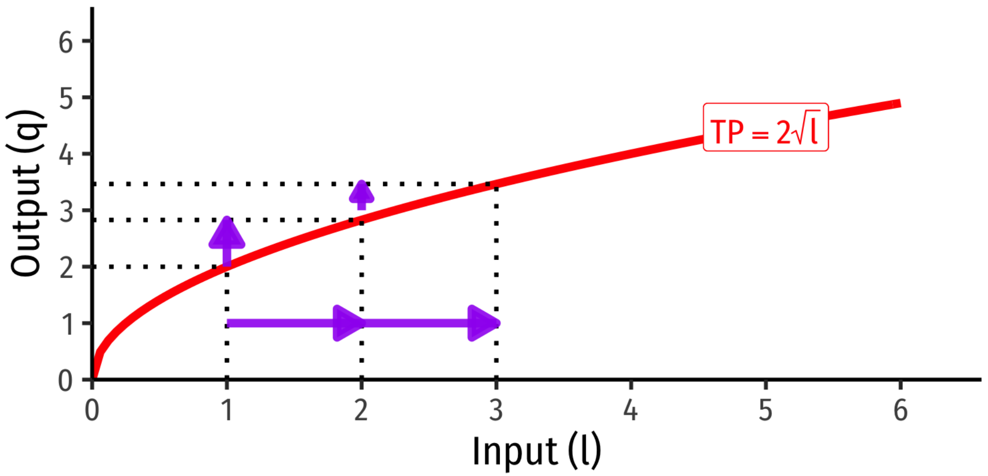


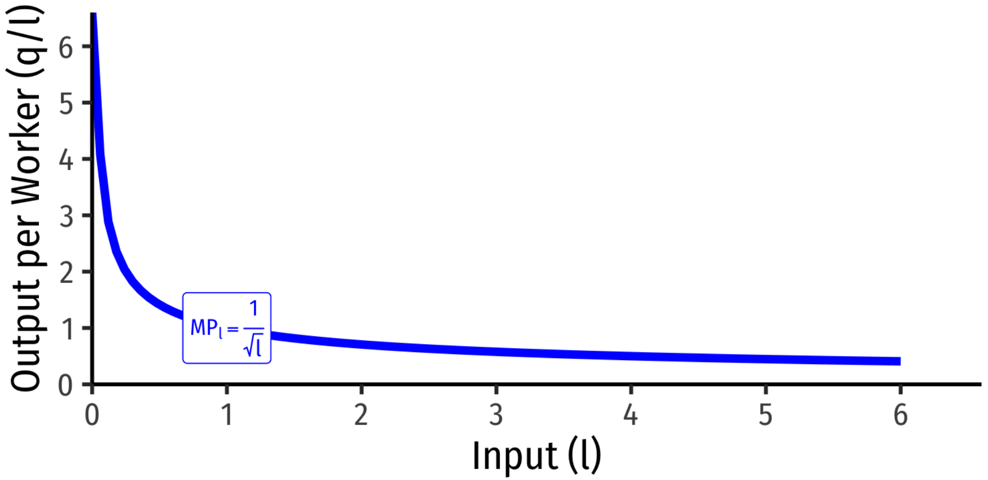


Diminishing Returns

- **Law of Diminishing Returns**: adding more of one factor of production **holding all others constant** will result in successively lower increases in output

- In order to increase output, firm will need to increase *all* factors!





Production Functions and Marginal Product

- A quick trick to roughly[†] estimate MP_l

$$MP_l \approx \frac{q_2 - q_1}{l_2 - l_1}$$

l	q	MP_l
0	0.00	–
1	2.00	2.00 – 0.00 = 2.00
2	2.83	2.83 – 2.00 = 0.83
3	3.46	3.46 – 2.83 = 0.63

[†] Note these are *approximate*. Technically, MP_l is defined via calculus as an *infinitesimal* change in l, whereas these are discrete changes.

Average Product of Labor (and Capital)

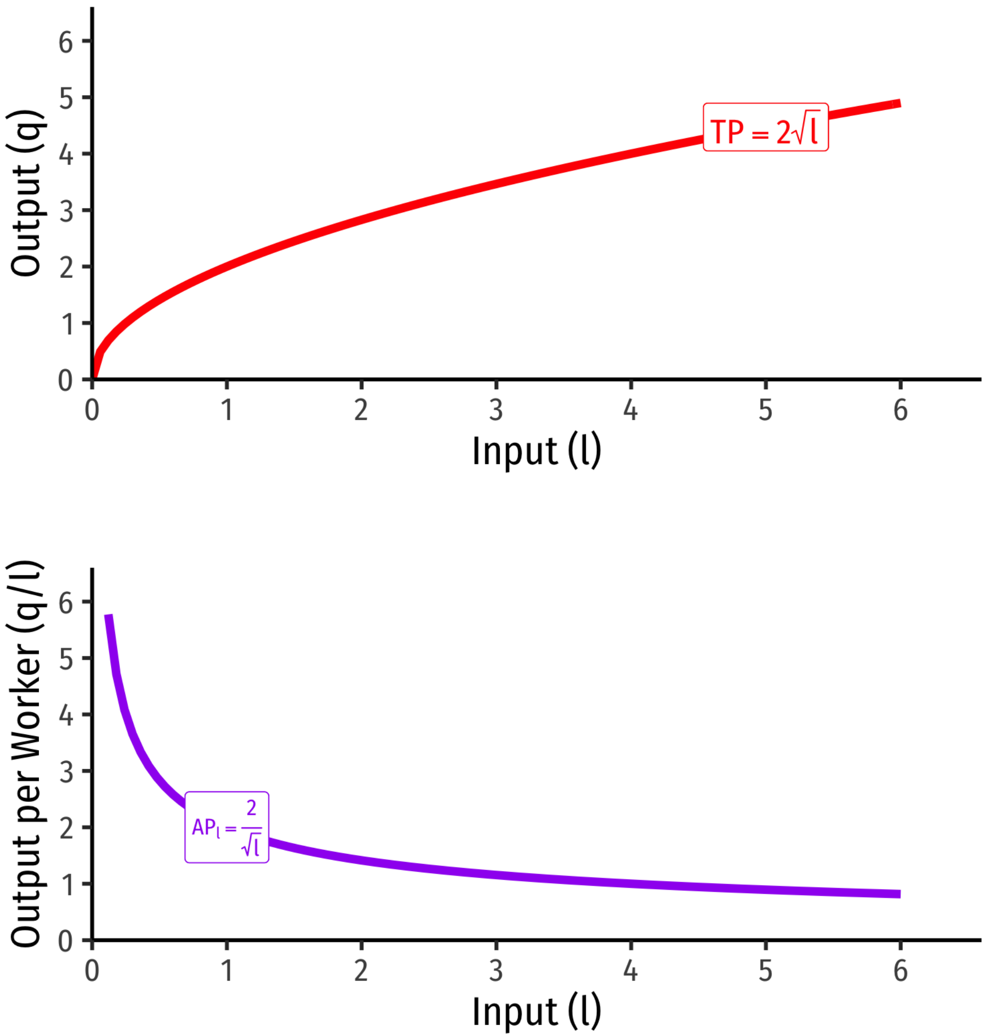
- **Average product of labor** (AP_l): total output per worker

$$AP_l = \frac{q}{l}$$

- A measure of *labor productivity*

- **Average product of capital** (AP_k): total output per unit of capital

$$AP_k = \frac{q}{k}$$



Production in the Short Run: Example II

Example: Suppose a firm has the following production function:

$$q = 2k + l^2$$

- Suppose in the short run, the firm has 10 units of capital.

1. Write an equation for the short run production function.
2. Calculate the total product(s), marginal product(s), and average product(s) for each of the first 5 workers.

The Firm's Problem: Long Run

The Long Run

- In the long run, *all* factors of production are **variable**

$$q = f(k, l)$$

- Can build more factories, open more storefronts, rent more space, invest in machines, etc.

- So the firm can choose both l *and* k

The Firm's Problem

- Based on what we've discussed, we can fill in a constrained optimization model for the firm
  - But don't write this one down just yet!
- The **firm's problem** is:

1. **Choose:** < inputs and output >
2. **In order to maximize:** < profits >
3. **Subject to:** < technology >

- It's actually much easier to break this into **2 stages**. See today's class notes page for an example using only one stage.

The Firm's Two Problems

1st Stage: firm's profit maximization problem:

1. **Choose:** **< output >**

2. **In order to maximize:** **< profits >**

- We'll cover this later...first we'll explore:

The Firm's Two Problems

1^{st} Stage: firm's profit maximization problem:

1. **Choose:** < output >
2. **In order to maximize:** < profits >

- We'll cover this later...first we'll explore:

2^{nd} Stage: firm's cost minimization problem:

1. **Choose:** < inputs >
2. **In order to *minimize*:** < cost >
3. **Subject to:** < producing the optimal output >

- Minimizing costs $\iff$ maximizing profits

Long Run Production

Example: $q = \sqrt{lk}$

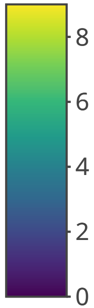
Capital, k (columns); Labor, l (rows)

	0	1	2	3	4	5
0	0.00	0.00	0.00	0.00	0.00	0.00
1	0.00	1.00	1.41	1.73	2.00	2.24
2	0.00	1.41	2.00	2.45	2.83	3.16
3	0.00	1.73	2.45	3.00	3.16	3.46
4	0.00	2.00	2.83	3.46	4.00	4.47
5	0.00	2.24	3.16	3.87	4.47	5.00

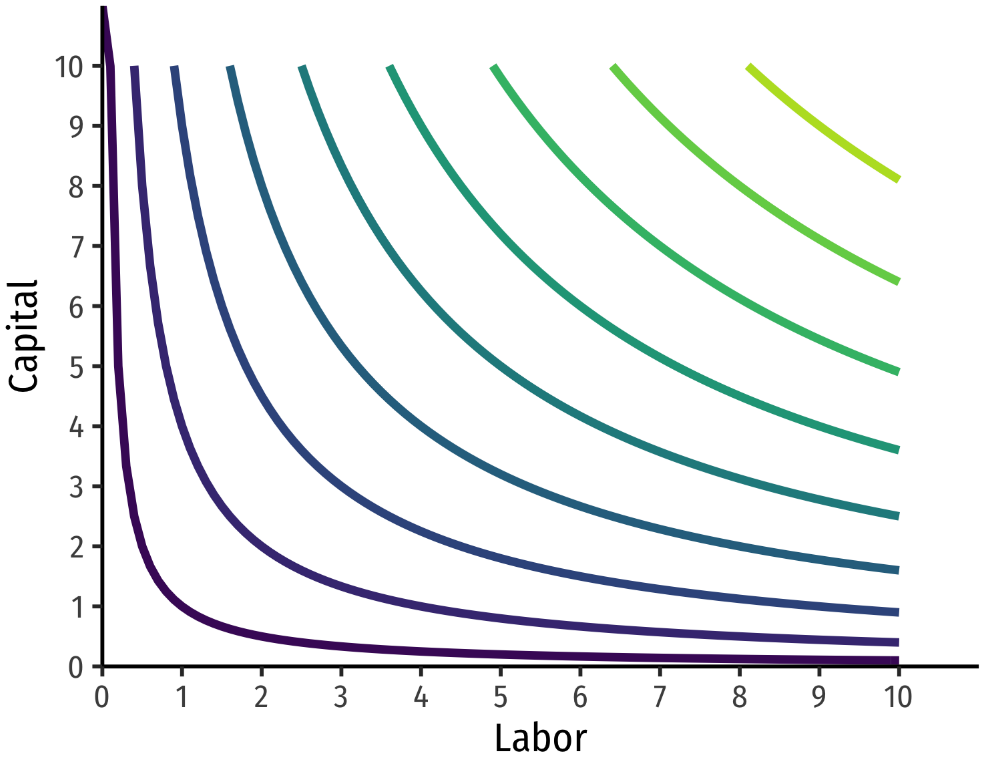
- Many input-combinations yield the same output!
- So how does the firm choose the *optimal* combination?

Mapping Input-Combination Choices Graphically

3-D Production Function



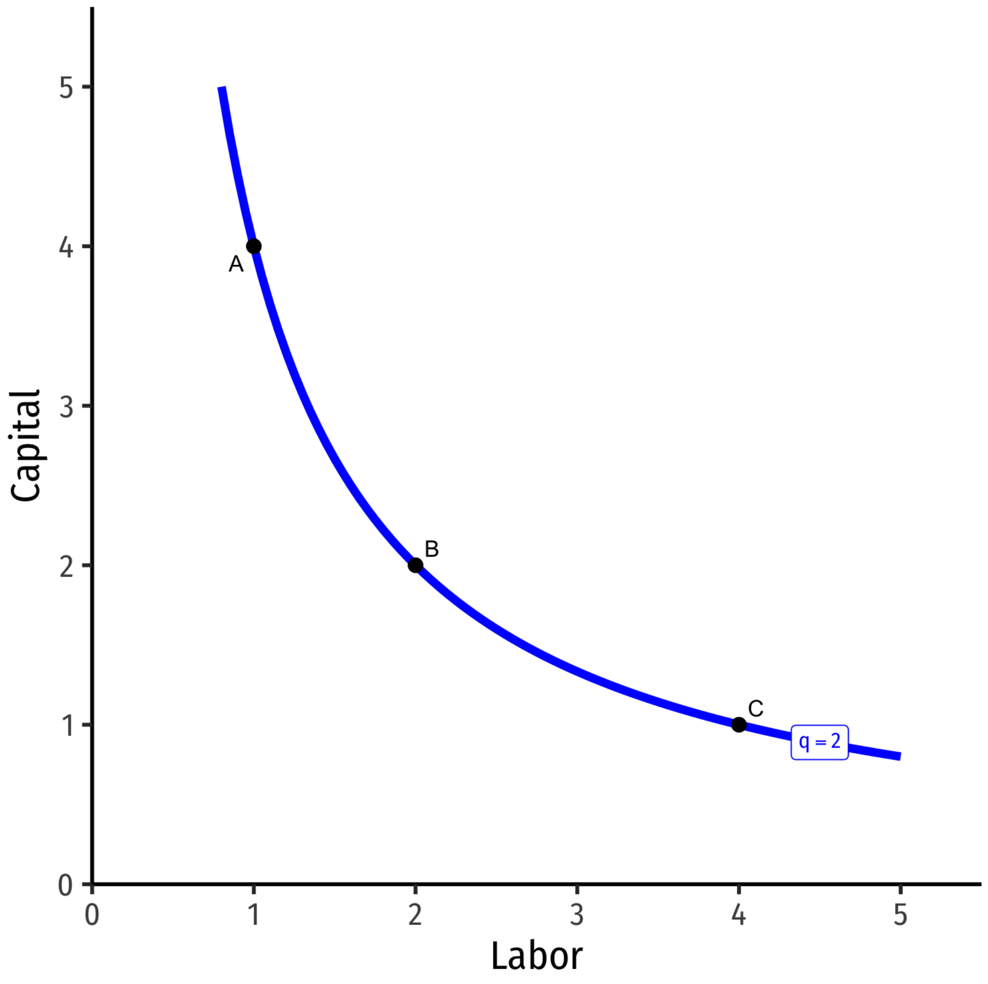
2-D Isoquant Contours



Isoquants and MRTS

Isoquant Curves

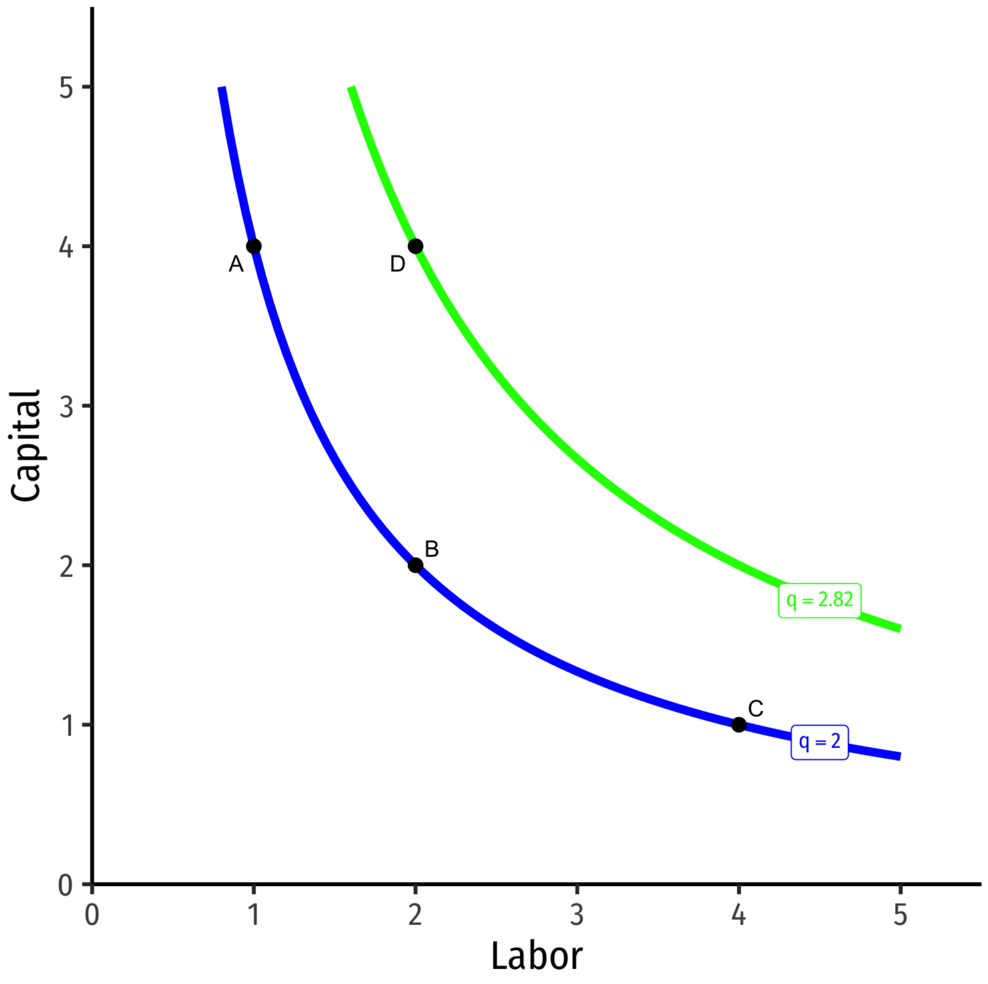
- We can draw an **isoquant** indicating all combinations of l and k that yield the same q



Isoquant Curves

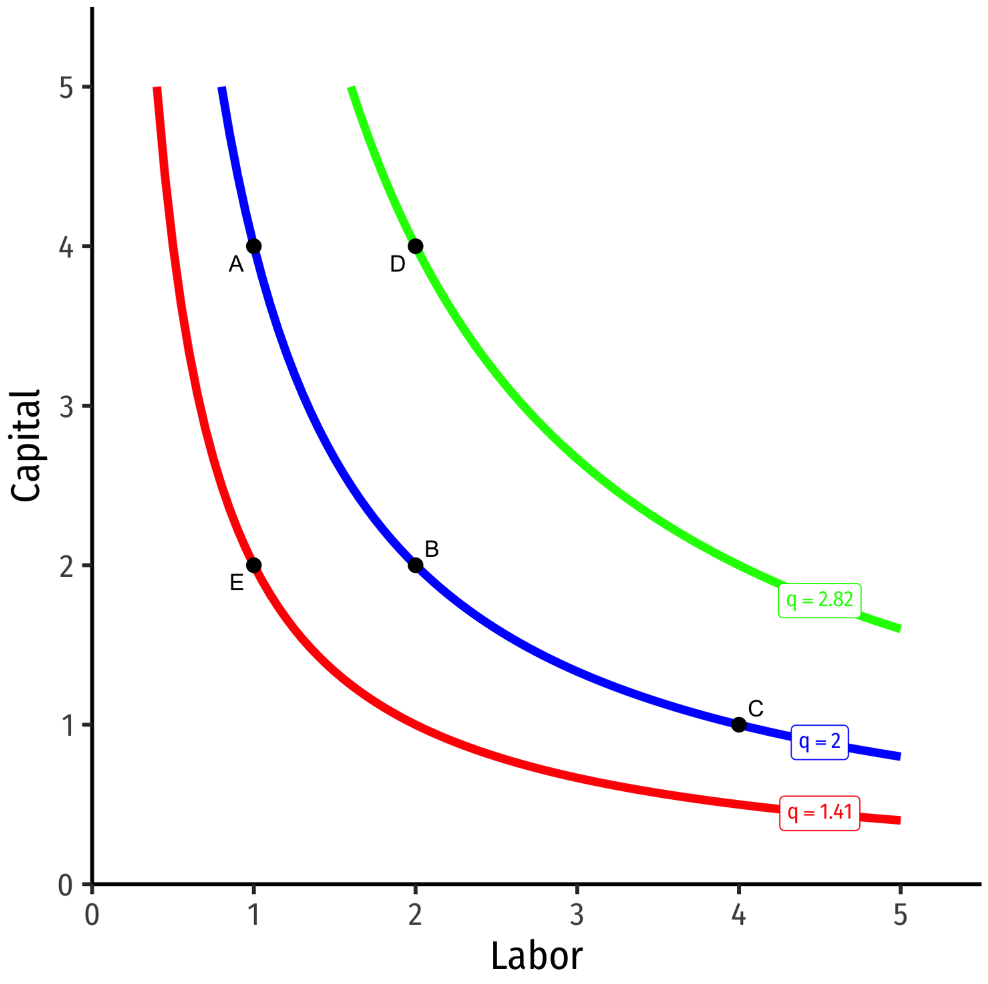
- We can draw an **isoquant** indicating all combinations of l and k that yield the same q

- Combinations *above* curve yield **more output**; on a **higher curve**
  - $D > A = B = C$



Isoquant Curves

- We can draw an **isoquant** indicating all combinations of l and k that yield the same q
- Combinations *above* curve yield **more output**; on a **higher curve**
  - $D > A = B = C$
- Combinations *below* the curve yield **less output**; on a **lower curve**
  - $E < A = B = C$



Marginal Rate of *Technical* Substitution I

- If your firm uses fewer workers, how much more capital would it need to produce the same amount?

Marginal Rate of *Technical* Substitution I

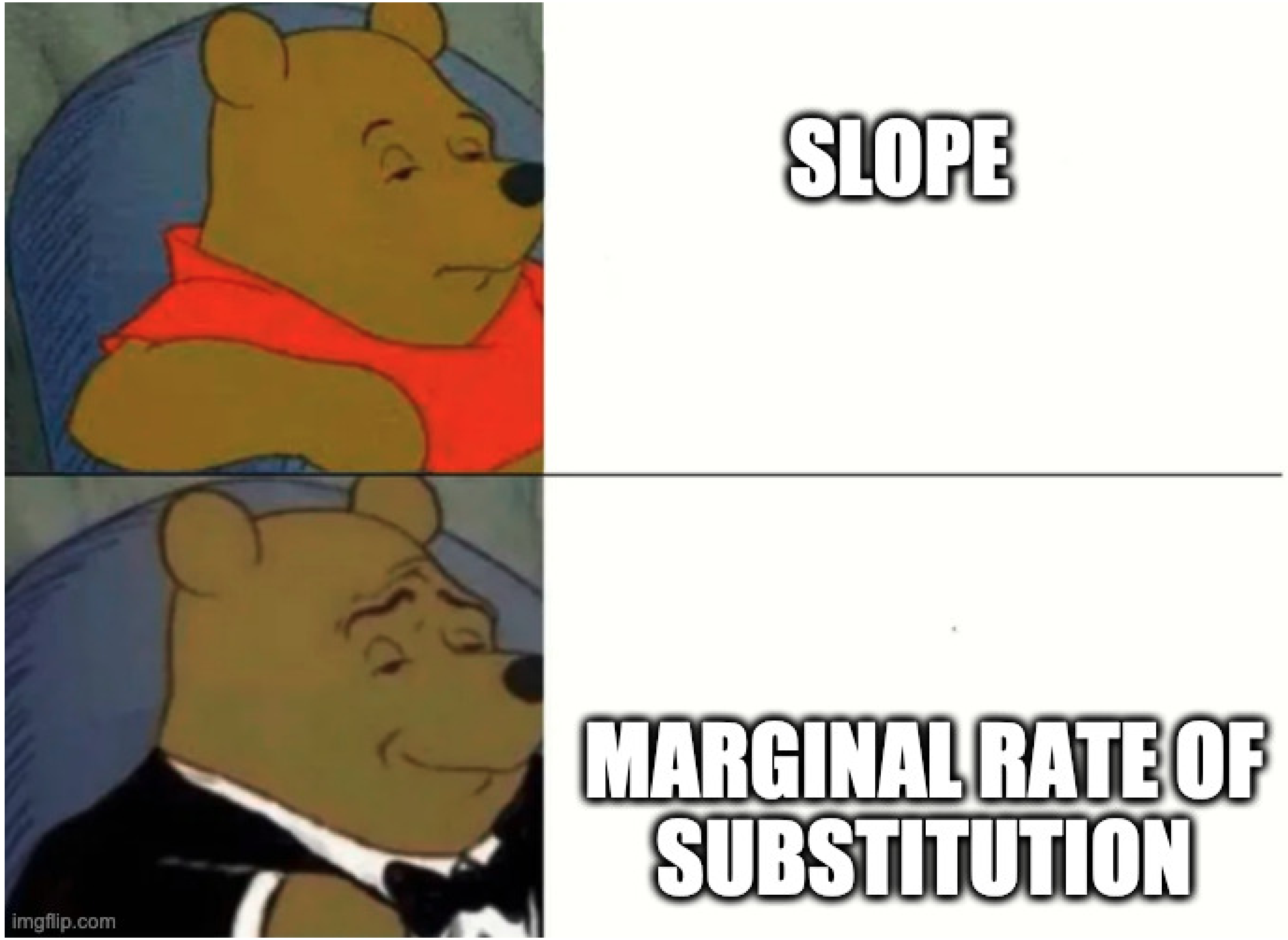
- If your firm uses fewer workers, how much more capital would it need to produce the same amount?

- **Marginal Rate of Technical Substitution (MRTS)**: rate at which firm trades off one input for another to *yield same output*

- Firm's **relative value** of using l in production based on its tech:

> "We could give up (MRTS) units of k to use 1 more unit of l to produce the same output."

Marginal Rate of *Technical* Substitution II

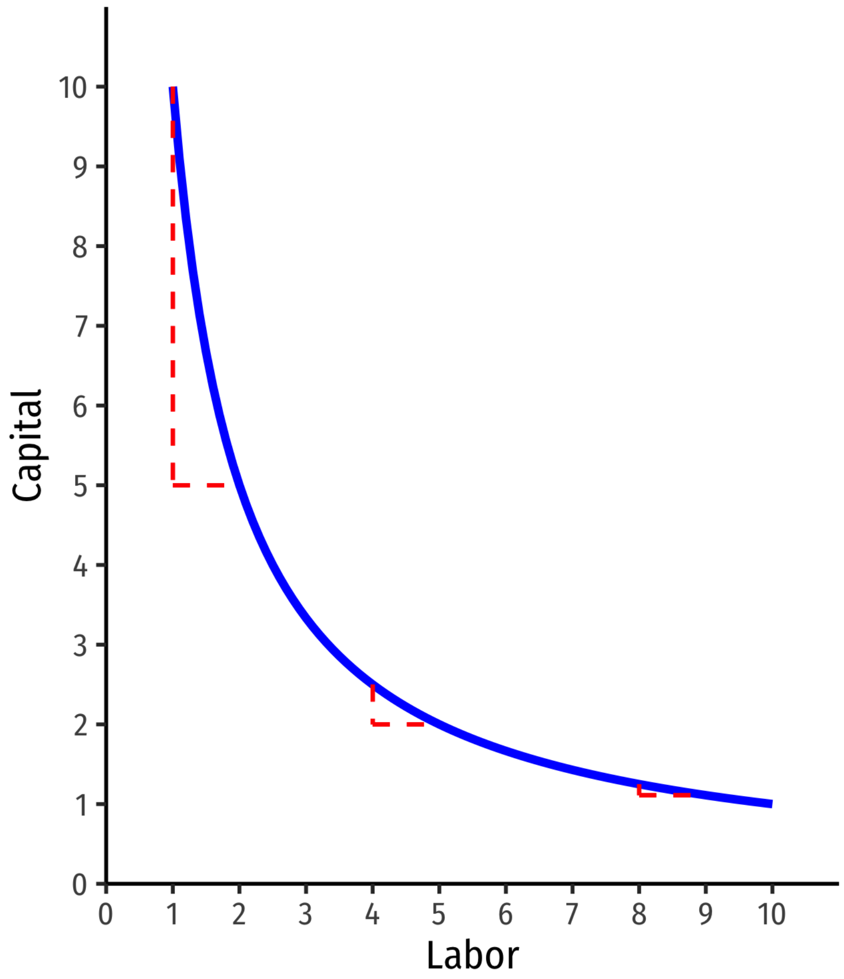


Marginal Rate of *Technical* Substitution II

- MRTS is the slope of the isoquant

$$MRTS_{l,k} = -\frac{\Delta k}{\Delta l} = \frac{rise}{run}$$

- Amount of k given up for 1 more l
- Note: slope (MRTS) changes along the curve!
- **Law of diminishing returns**!

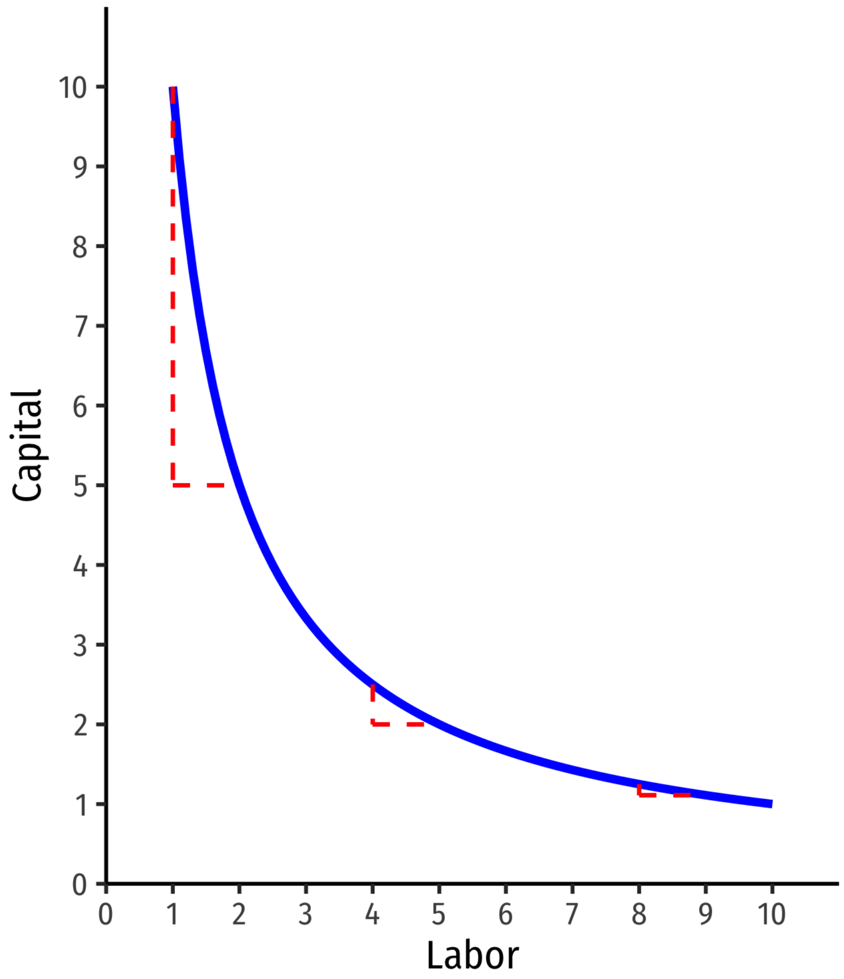


MRTS and Marginal Products

- Relationship between MP and $MRTS$:

$$\underbrace{\frac{\Delta k}{\Delta l}}_{MRTS} = -\frac{MP_l}{MP_k}$$

- See proof in [today's class notes]
- Sound familiar? 🧐



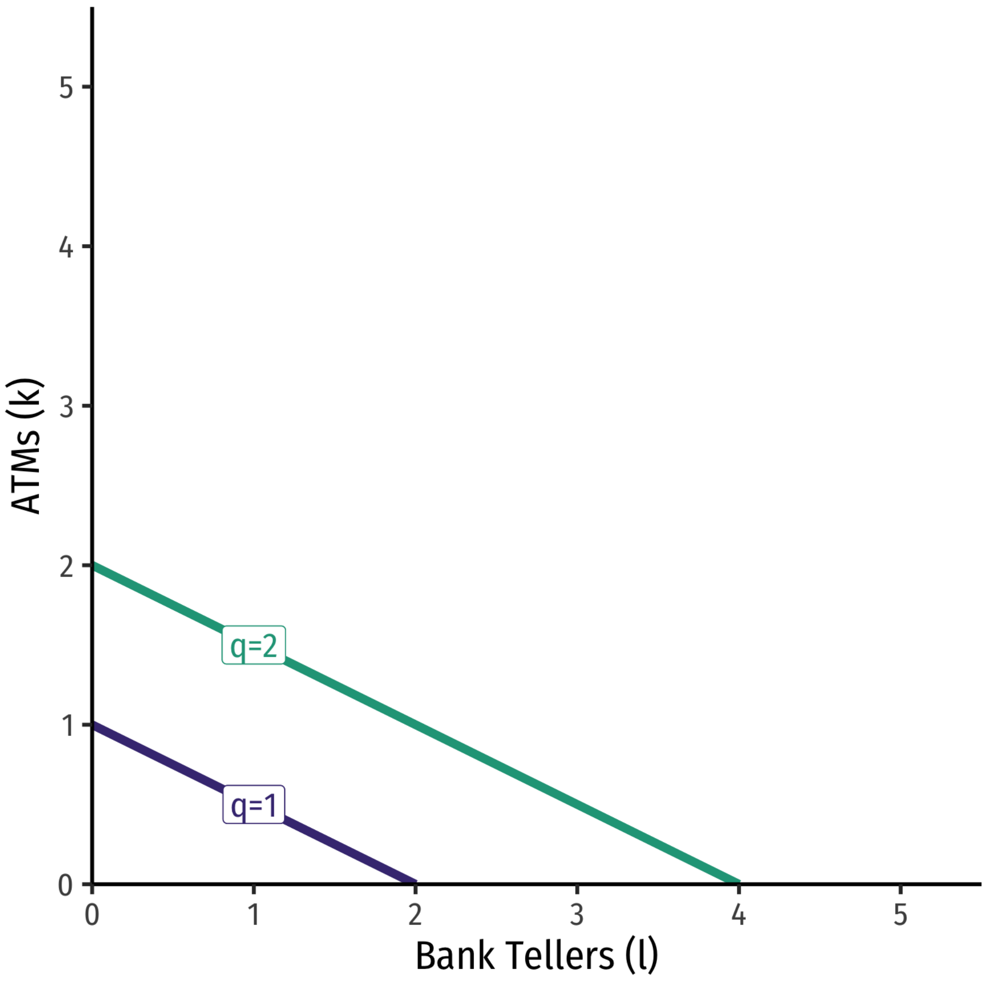
Special Case I: Perfect Substitutes

Example: Consider Bank Tellers (l) and ATMs (k)

- Suppose 1 ATM can do the work of 2 bank tellers

- **Perfect substitutes**: inputs that can be substituted at same fixed rate and yield same output

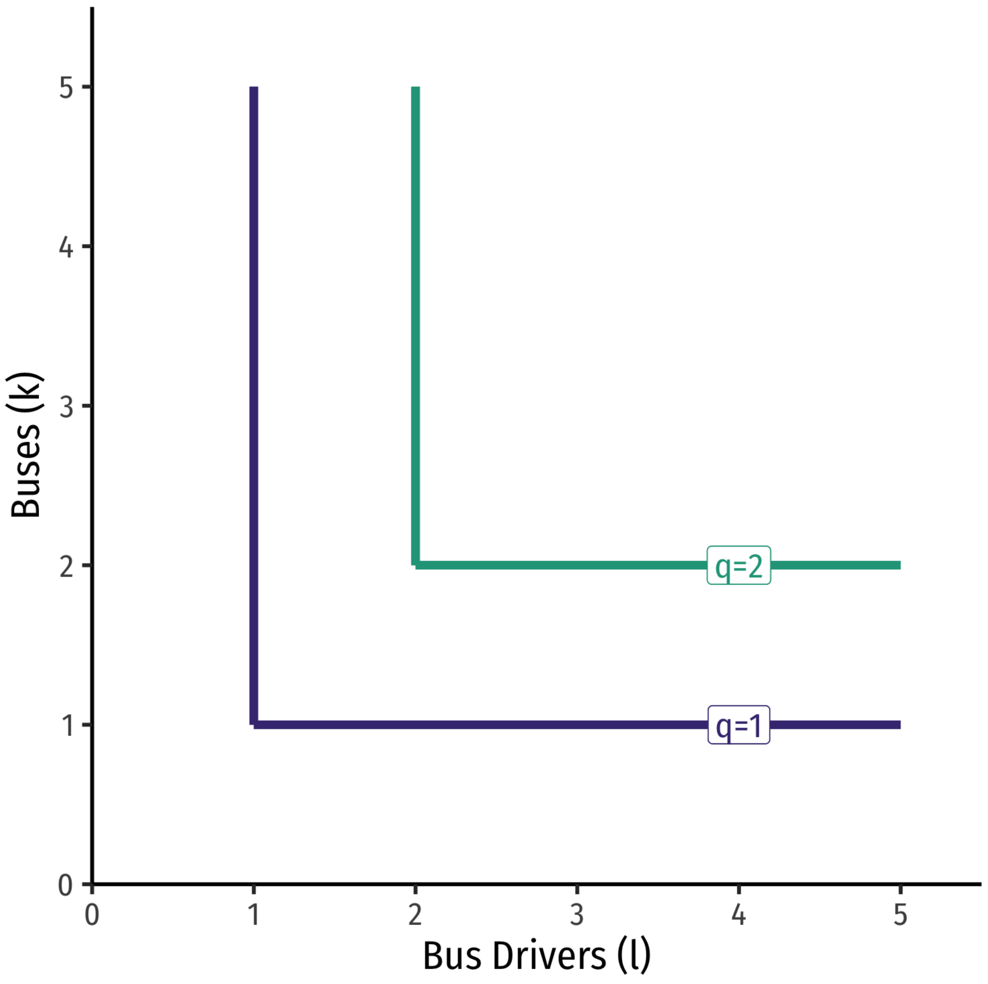
- $MRTS_{l,k} = -0.5$ (a constant!)



Special Case II: Perfect Complements

Example: Consider buses (k) and bus drivers (l)

- Must combine together in fixed proportions (1:1)
- **Perfect complements**: inputs must be used together in same fixed proportion to produce output
- $MRTS_{l,k}$: ?

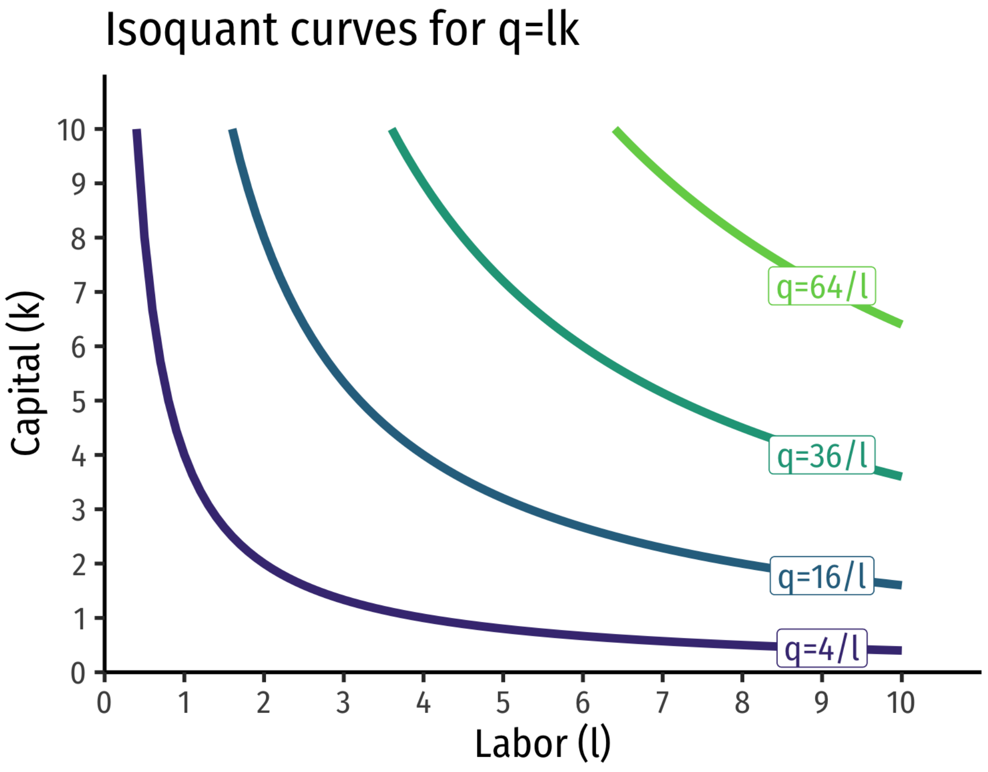


Common Case: Cobb-Douglas Production Functions

- Again: very common functional form in economics is **Cobb-Douglas**

$$q = A\,k^a l^b$$

- Where $a, b > 0$
  - often $a + b = 1$
- A is total factor productivity



Practice

Example: Suppose a firm has the following production function:

$$q = 2lk$$

Where its marginal products are:

$$\begin{aligned} MP_l &= 2k \\ MP_k &= 2l \end{aligned}$$

1. Put l on the horizontal axis and k on the vertical axis. Write an equation for $MRTS_{l,k}$.
2. Would input combinations of $(1, 4)$ and $(2, 2)$ be on the same isoquant?
3. Sketch a graph of the isoquant from part 2.

Isocost Lines

Isocost Lines

- If your firm can choose among *many* input combinations to produce q, which combinations are optimal?

- Those combination that are **cheapest**

- Denote prices of each input as:
  - w: price of labor (wage)
  - r: price of capital

- Let C be **total cost** of using inputs (l, k) at market prices (w, r) to produce q units of output:

$$C(w, r, q) = wl + rk$$

The Isocost Line, Graphically

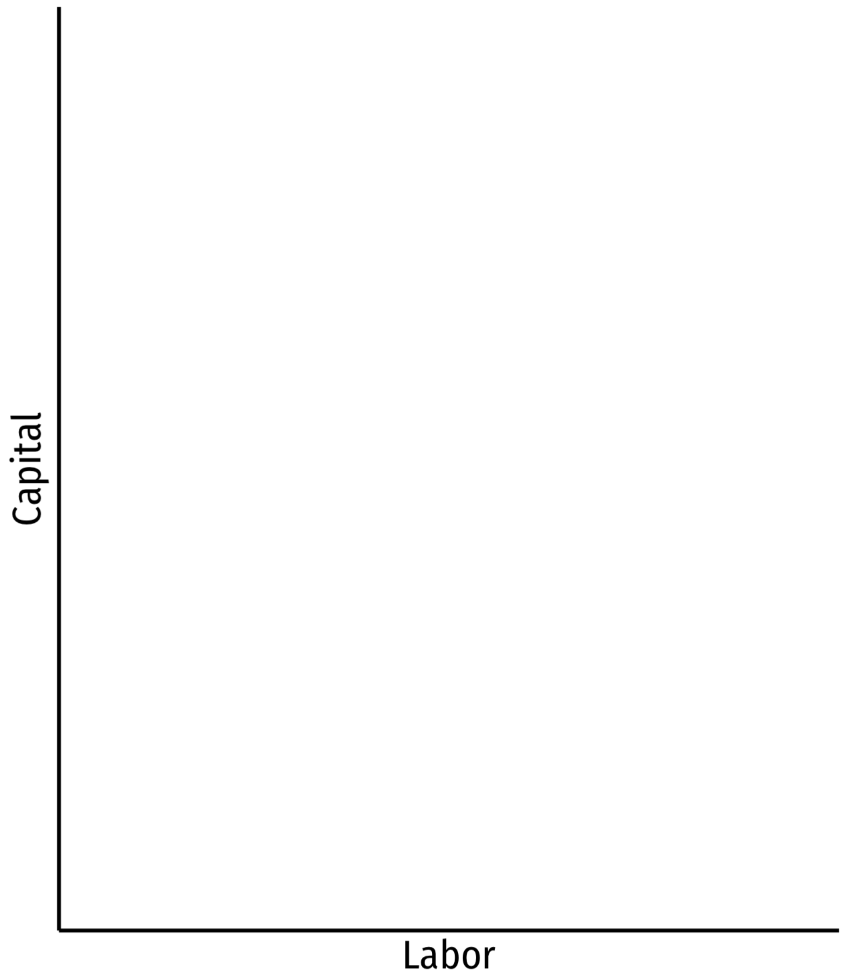
$$wl + rk = C$$

The Isocost Line, Graphically

$$wl + rk = C$$

- Solve for k to graph

$$k = \frac{C}{r} - \frac{w}{r}l$$



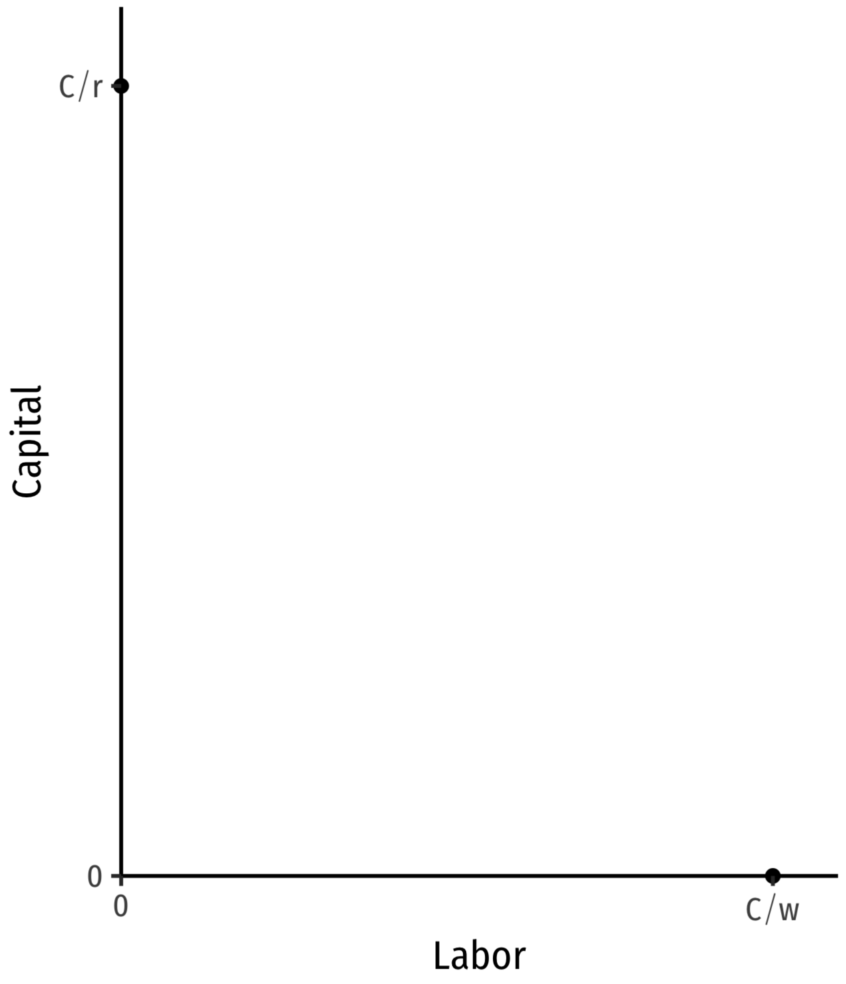
The Isocost Line, Graphically

$$wl + rk = C$$

- Solve for k to graph

$$k = \frac{C}{r} - \frac{w}{r}l$$

- Vertical-intercept: $\frac{C}{r}$
- Horizontal-intercept: $\frac{C}{w}$



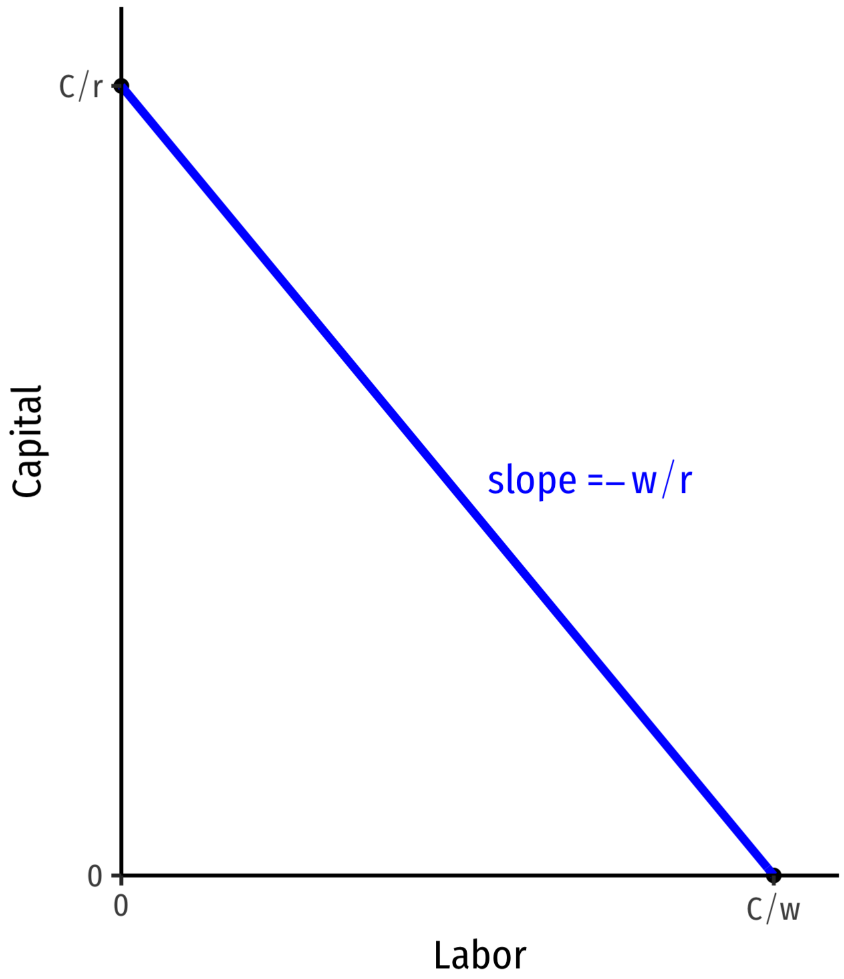
The Isocost Line, Graphically

$$wl + rk = C$$

- Solve for k to graph

$$k = \frac{C}{r} - \frac{w}{r}l$$

- Vertical-intercept: $\frac{C}{r}$
- Horizontal-intercept: $\frac{C}{w}$
- slope: $-\frac{w}{r}$



The Isocost Line: Example

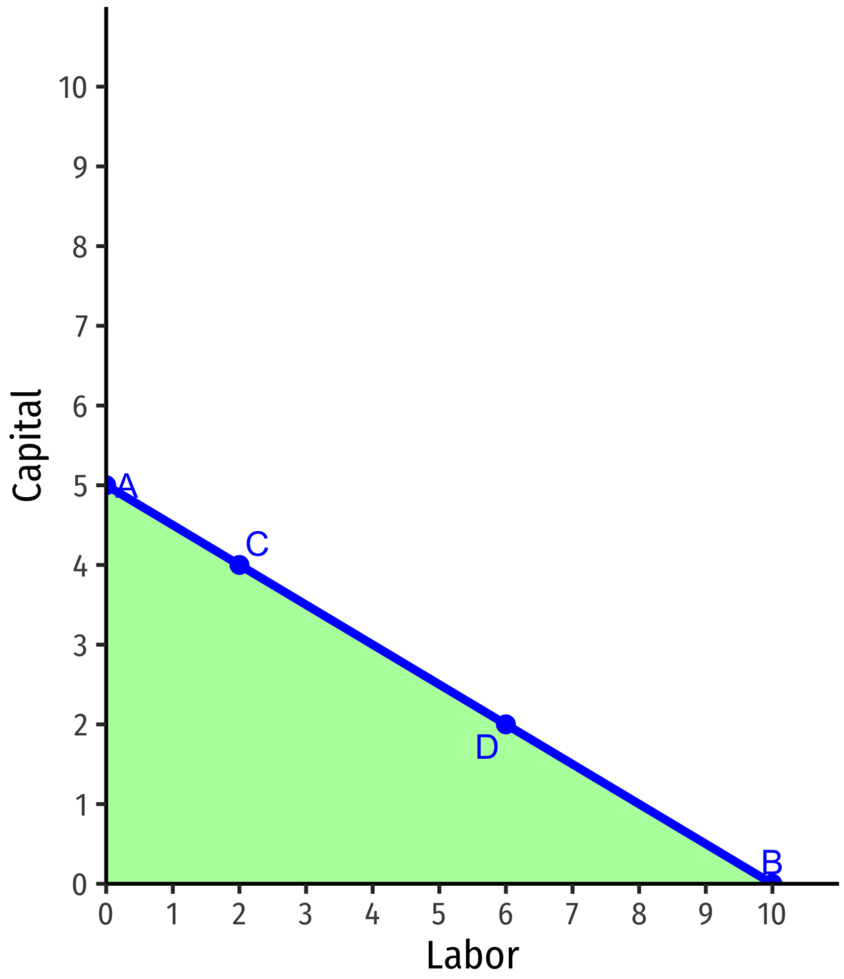
Example: Suppose your firm has a purchasing budget of \$50. Market wages are \$5/worker-hour and the mark rental rate of capital is \$10/machine-hour. Let l be on the horizontal axis and k be on the vertical axis.

1. Write an equation for the isocost line (in graphable form).

2. Graph the isocost line.

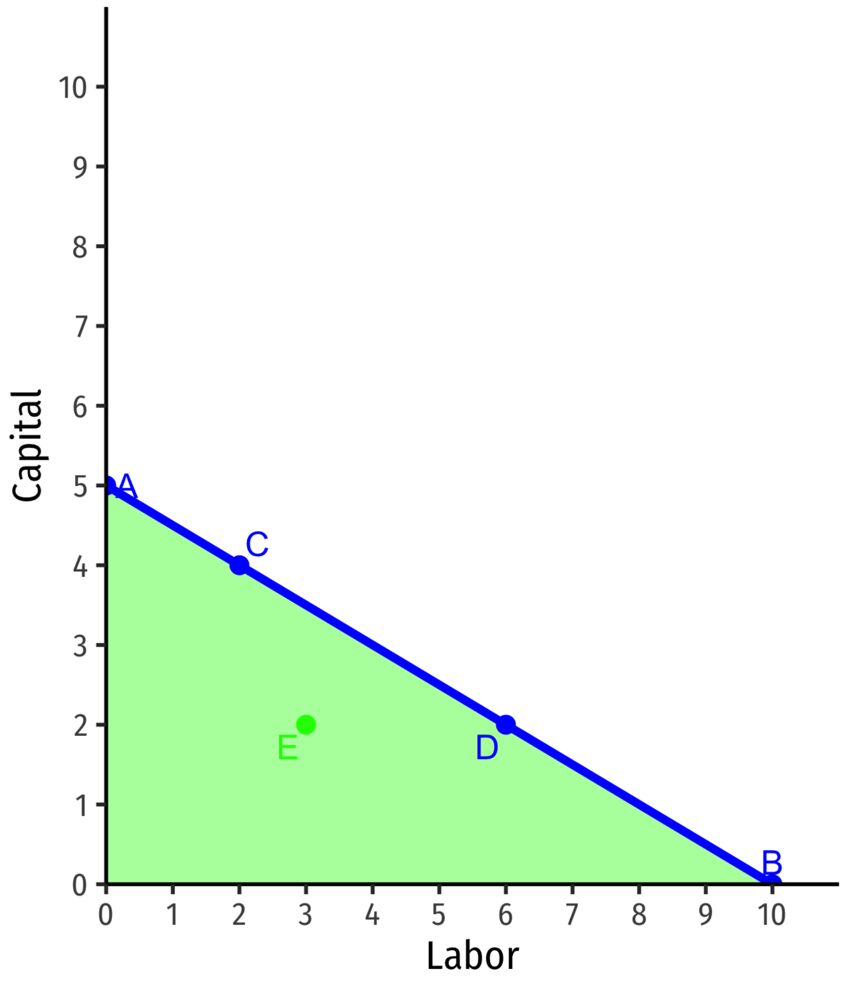
Interpreting the Isocost Line

- Points on the line are same total cost
  - A: $\$5(0l) + \$10(5k) = \$50$
  - B: $\$5(10l) + \$10(0k) = \$50$
  - C: $\$5(2l) + \$10(4k) = \$50$
  - D: $\$5(6l) + \$10(2k) = \$50$



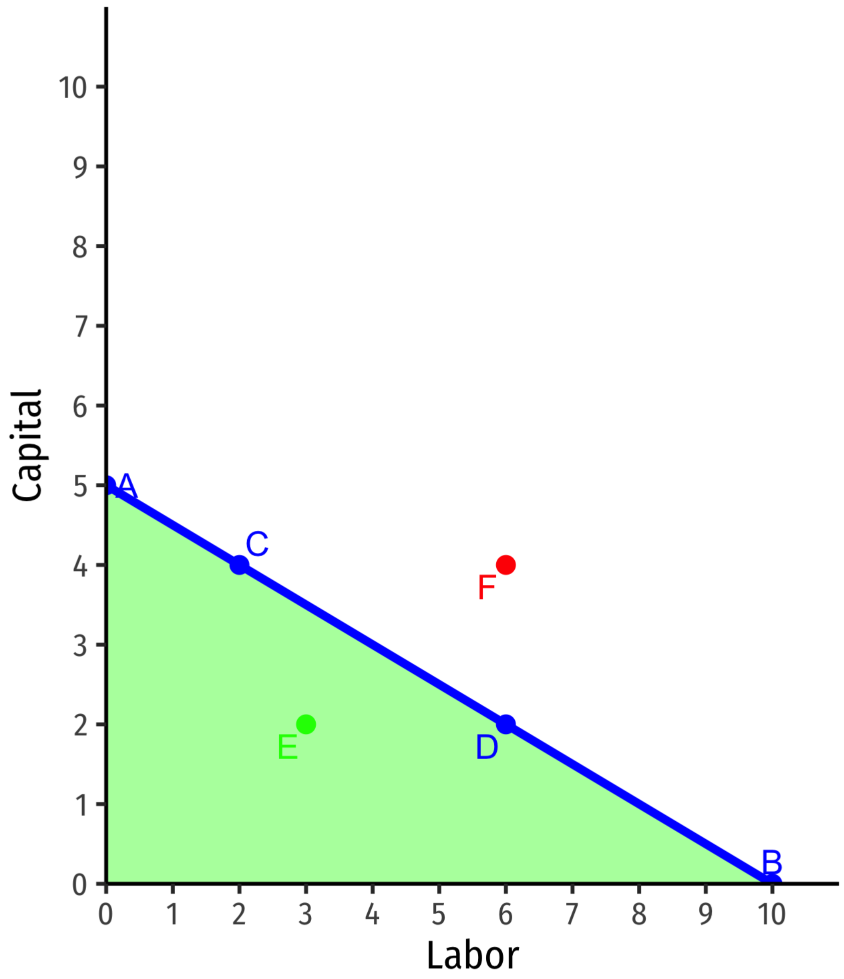
Interpreting the Isocost Line

- Points on the line are same total cost
  - A: $\$5(0l)+\$10(5k)=\$50$
  - B: $\$5(10l)+\$10(0k)=\$50$
  - C: $\$5(2l)+\$10(4k)=\$50$
  - D: $\$5(6l)+\$10(2k)=\$50$
- Points beneath the line are cheaper (but may produce less)
  - E: $\$5(3l)+\$10(2k)=\$35$



Interpreting the Isocost Line

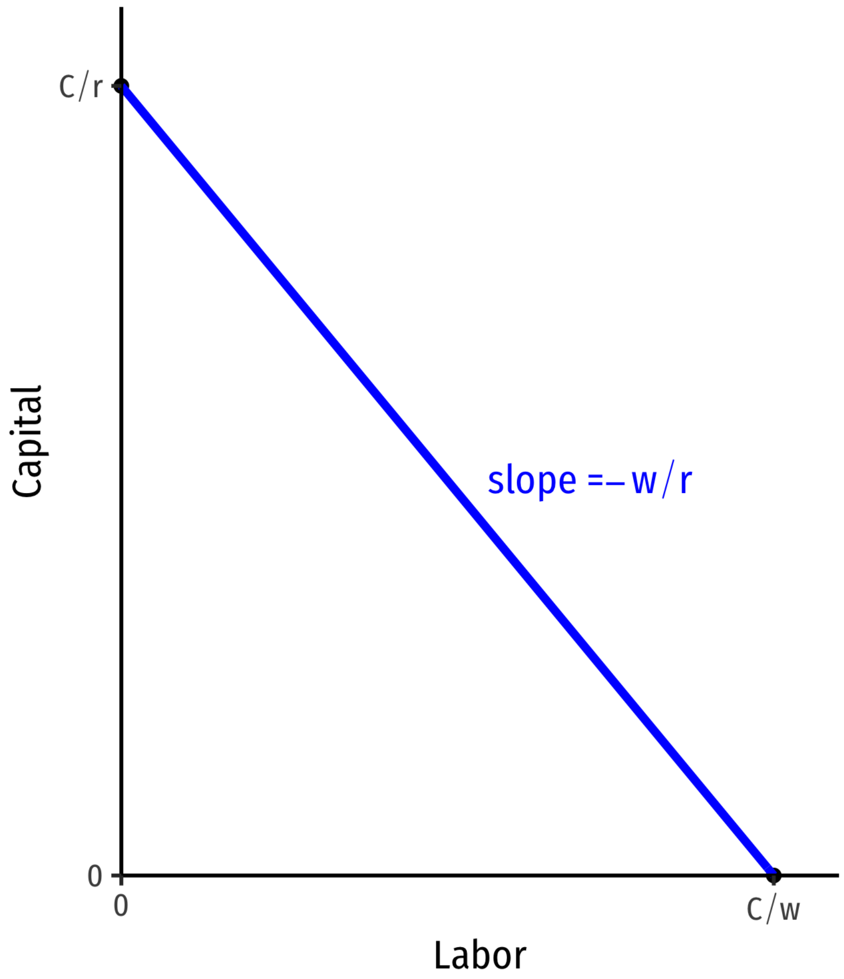
- Points on the line are same total cost
  - A: $\$5(0l) + \$10(5k) = \$50$
  - B: $\$5(10l) + \$10(0k) = \$50$
  - C: $\$5(2l) + \$10(4k) = \$50$
  - D: $\$5(6l) + \$10(2k) = \$50$
- Points beneath the line are cheaper (but may produce less)
  - E: $\$5(3l) + \$10(2k) = \$35$
- Points above the line are more expensive (and may produce more)
  - F: $\$5(6l) + \$10(4k) = \$70$



Interpretting the Slope

- **Slope**: **tradeoff** between l and k at market prices
  - Market **"exchange rate"** between l and k
- ***Relative* price** of l or the **opportunity cost** of l:

> Hiring 1 more unit of l requires giving up $\left(\frac{w}{r}\right)$ units of k

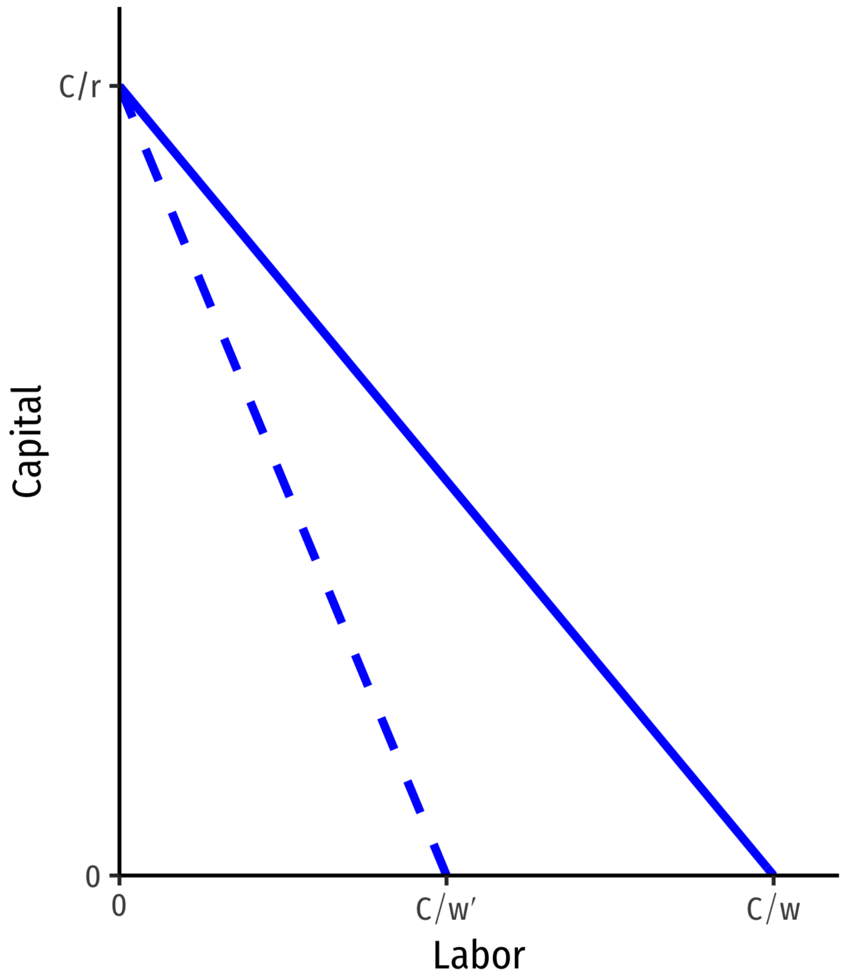


Changes in Relative Factor Prices I

- Changes in **relative factor prices**: *rotate* the line

Example: An increase in the price of l

- Slope changes: $-\frac{w'}{r}$



Changes in Relative Factor Prices II

- Changes in **relative factor prices**: *rotate* the line

Example: An increase in the price of k

- Slope changes: $-\frac{w}{r'}$