

1.6 — Income & Substitution Effects

ECON 306 • Microeconomic Analysis • Fall 2021

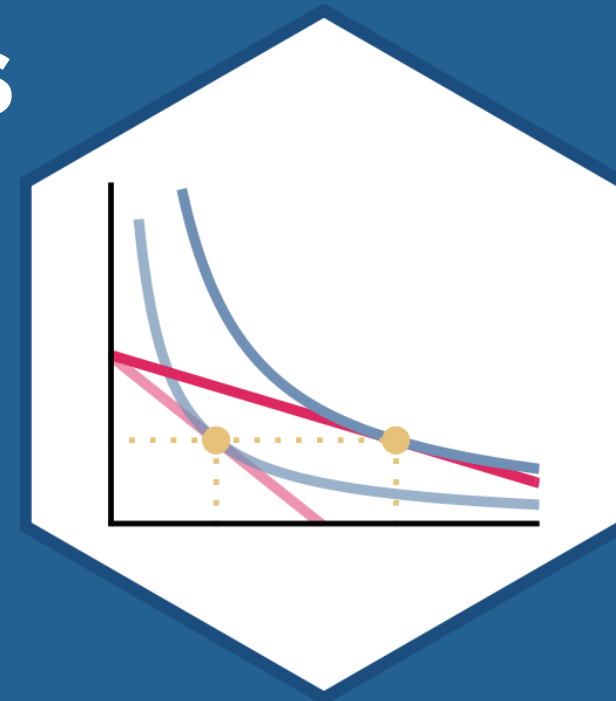
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Outline



The (Own) Price Effect

(Real) Income Effect

Substitution Effect

Putting the Effects Together

From Optimal Consumption Points to Demand

A Demand Function (Again)



- A consumer's **demand** (for good x) depends on current prices & income:

$$q_x^D = q_x^D(m, p_x, p_y)$$

- How does **demand for x** change?

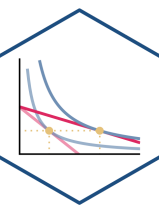
1. **Income effects** $\left(\frac{\Delta q_x^D}{\Delta m}\right)$: how q_x^D changes with changes in income
2. **Cross-price effects** $\left(\frac{\Delta q_x^D}{\Delta p_y}\right)$: how q_x^D changes with changes in prices of *other* goods (e.g. y)
3. **(Own) Price effects** $\left(\frac{\Delta q_x^D}{\Delta p_x}\right)$: how q_x^D changes with changes in price (of x)





The (Own) Price Effect

The (Own) Price Effect



- **Price effect:** change in optimal consumption of a good associated with a change in its price, holding income and other prices constant

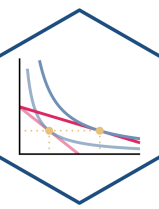
$$\frac{\Delta q_x^D}{\Delta p_x} < 0$$

The law of demand: as the price of a good rises, people will tend to buy less of that good (and vice versa)

- i.e. **the price effect is negative!**



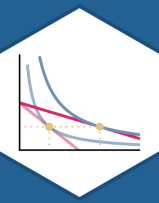
Decomposing the Price Effect



The **price effect** (law of demand) is actually the **net result of two effects**

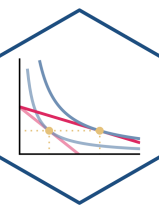
1. **(Real) income effect**: change in consumption due to change in real purchasing power
2. **Substitution effect**: change in consumption due to change in relative prices

$$\text{Price Effect} = \text{Real income effect} + \text{Substitution Effect}$$



(Real) Income Effect

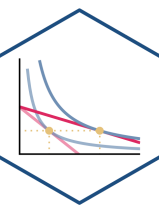
(Real) Income Effect: Demonstration



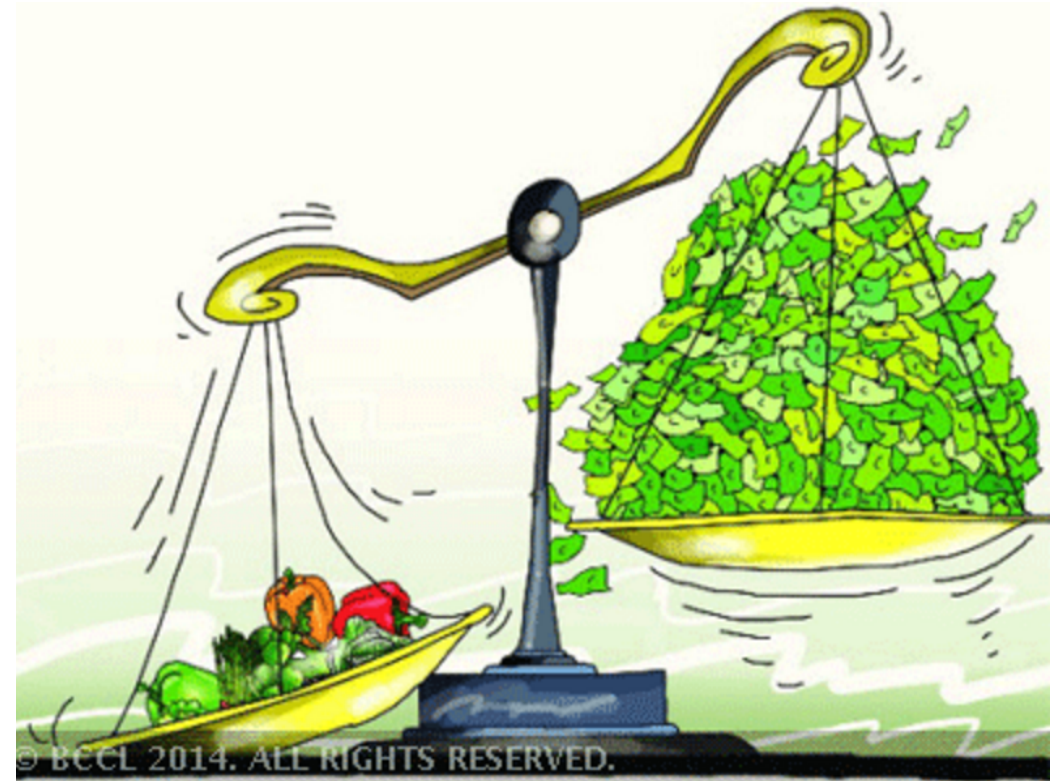
- Suppose there is only 1 good to consume, x . You have a \$100 income, and the price of x is \$10. You consume 10 units of x
- Suppose the price of x falls to \$5. Your now consume 20 units of x .
- This is the **real income effect**



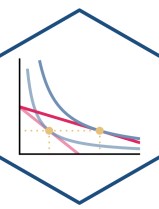
(Real) Income Effect: Demonstration



- **Real income effect:** your consumption mix changes because of the change in the price of x changes your **real income** or **purchasing power** (the amount of goods you can buy)
- Note your **actual (nominal) income** (\$100) **never changed!**



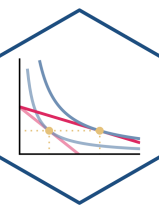
(Real) Income Effect: Size



- The *size* of the income effect depends on how large a *portion of your budget* you spend on the good
- **Large-budget items:**
 - e.g. Housing/apartment rent, car prices
 - Price increase makes you much poorer
 - Price decrease makes you much wealthier



(Real) Income Effect: Size



- The *size* of the income effect depends on how large a *portion of your budget* you spend on the good
- **Small-budget items:**
 - e.g. pencils, toothpicks, candy
 - Price changes don't have much of an effect on your wealth or change your behavior much





Substitution Effect

Substitution Effect: Demonstration



- Suppose there are 1000's of goods, none of them a major part of your budget
 - So real income effect is insignificant
- Suppose the price of one good, x increases
- You would consume *less* of x relative to other goods because x is now *relatively* more expensive
- That's the **substitution effect**

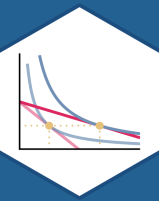


Substitution Effect: Demonstration



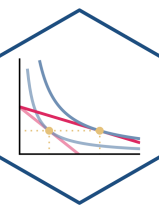
- **Substitution effect**: consumption mix changes because of a change in **relative prices**
- Buy more of the (now) relatively cheaper items
- Buy less of the (now) relatively more expensive item (x)





Putting the Effects Together

Putting the Effects Together



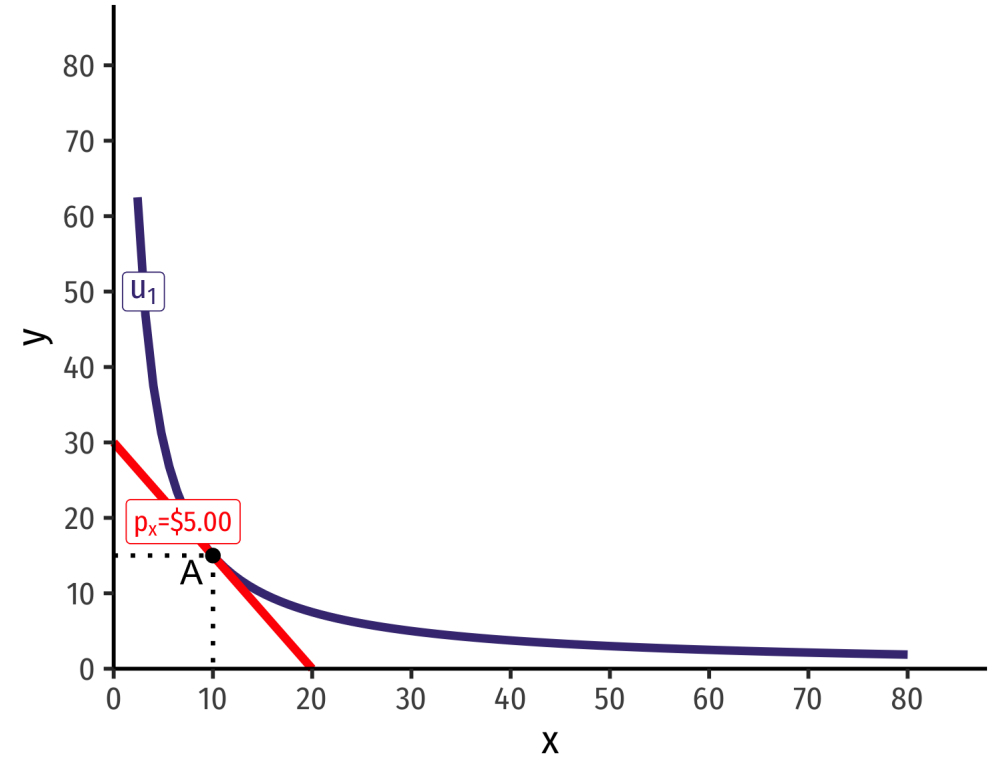
- **Real income effect:** change in consumption due to change in real purchasing power
 - Can be positive (**normal goods**) or negative (**inferior goods**)
 - Lower price of x means you can buy more x , y , or *both* (depending on your preferences between x and y)
- **Substitution effect:** change in consumption due to change in relative prices
 - If x gets cheaper relative to y , consume $\downarrow y$ (and $\uparrow x$)
 - This is always the same direction! (\downarrow relatively expensive goods, *uparrow* relatively cheaper goods)
 - This is why demand curves slope downwards!

$$\text{Price Effect} = \text{Real income effect} + \text{Substitution Effect}$$

Real Income and Substitution Effects, Graphically I

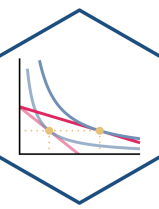


- Original optimal consumption (A)

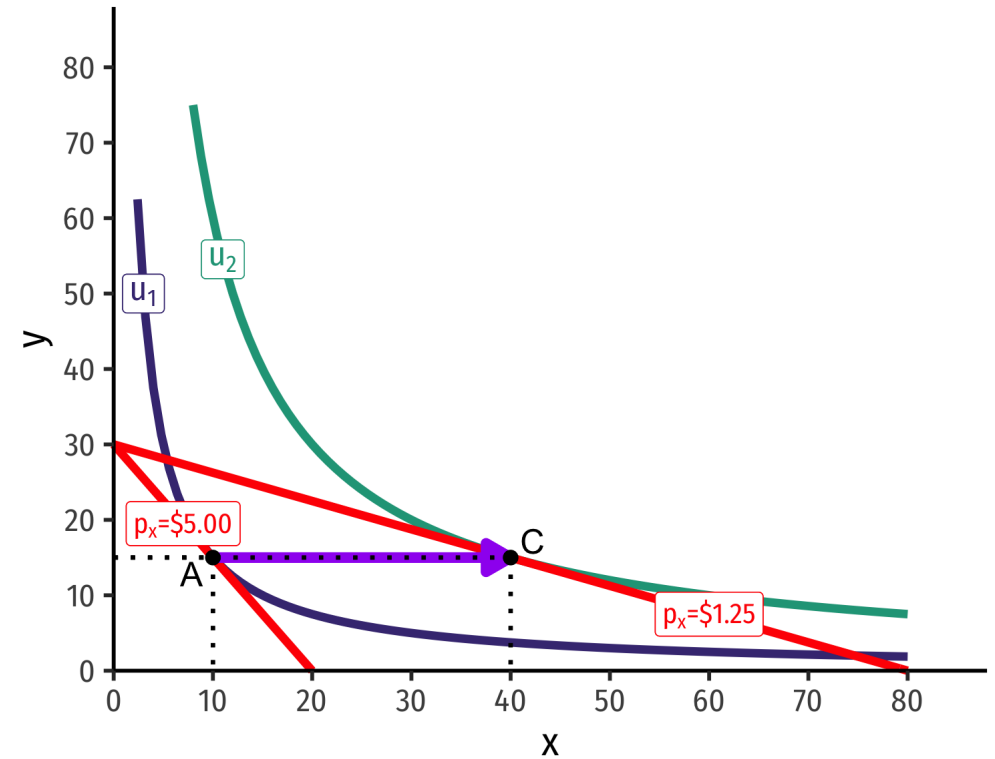


Optima with $u(x, y) = x^{0.5}y^{0.5}$, $m = 100$, $p_y = 3.33$

Real Income and Substitution Effects, Graphically I

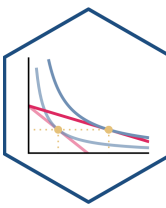


- Original optimal consumption (A)
- **(Total) price effect: $A \rightarrow C$**
- Let's decompose this into the two effects

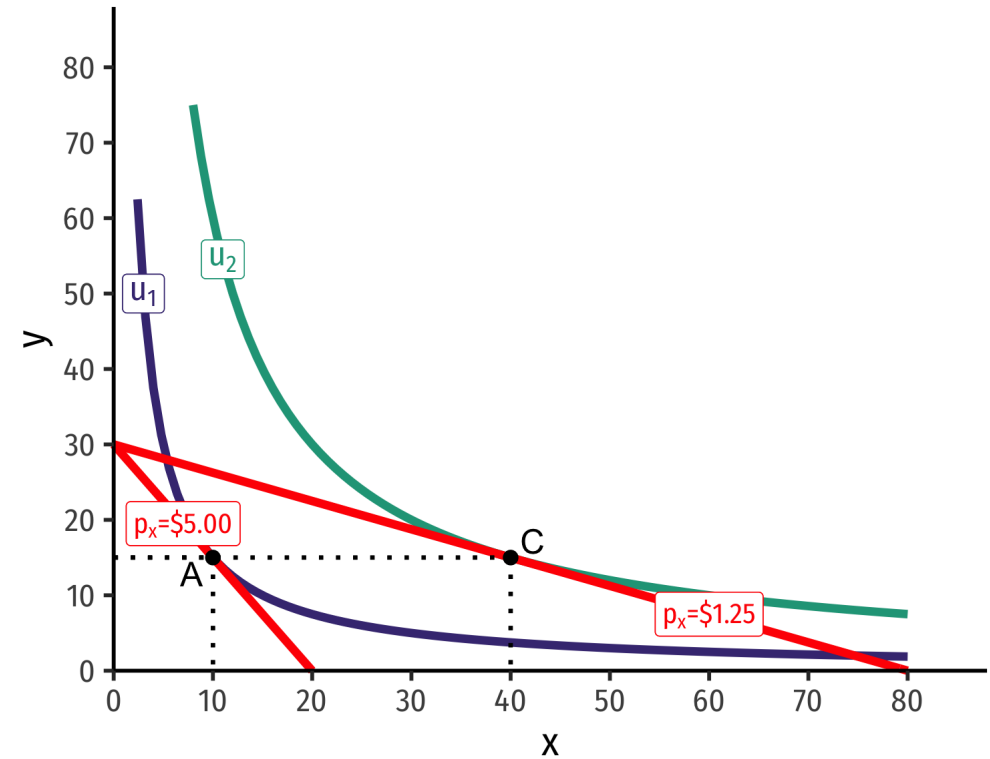


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Real Income and Substitution Effects, Graphically II

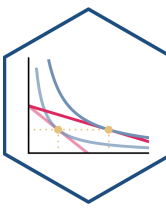


- **Substitution effect:** what you would choose under the **new exchange rate** to **remain indifferent** as before the change

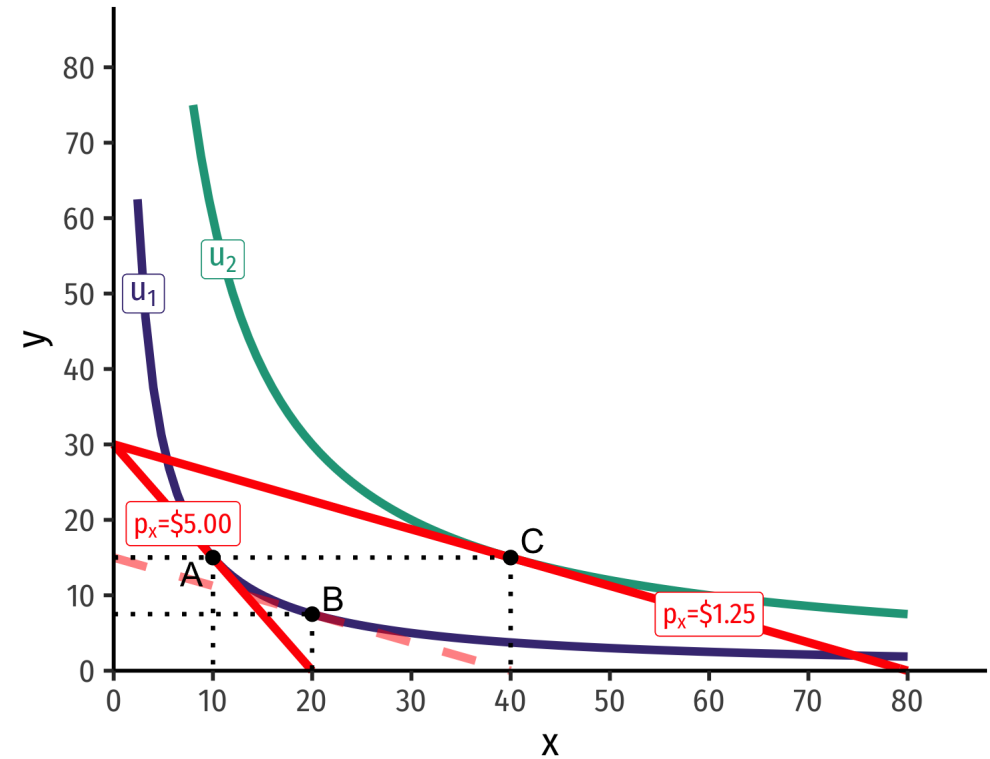


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Real Income and Substitution Effects, Graphically II

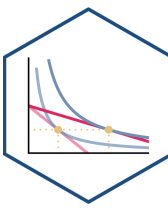


- **Substitution effect:** what you would choose under the **new exchange rate** to **remain indifferent** as before the change
- Graphically: shift *new* budget constraint inwards until tangent with *old* indifference curve
- $A \rightarrow B$ on same I.C. ($\uparrow x, \downarrow y$)
 - Point B *must* be a *different* point on the original curve!

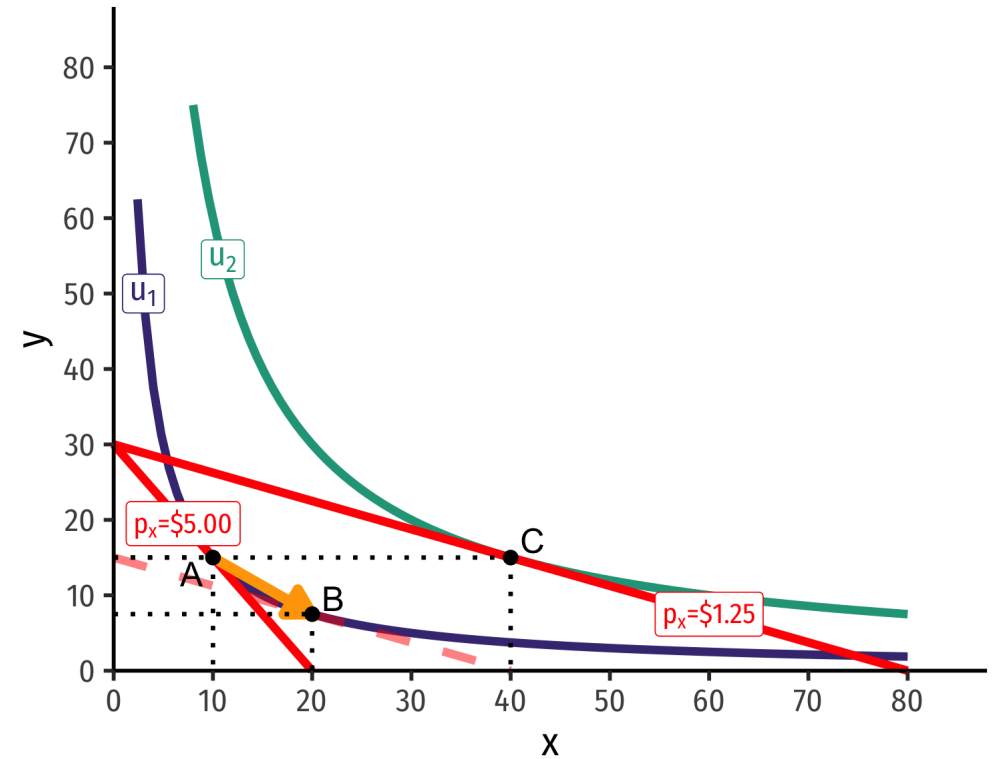


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Real Income and Substitution Effects, Graphically II

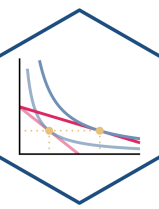


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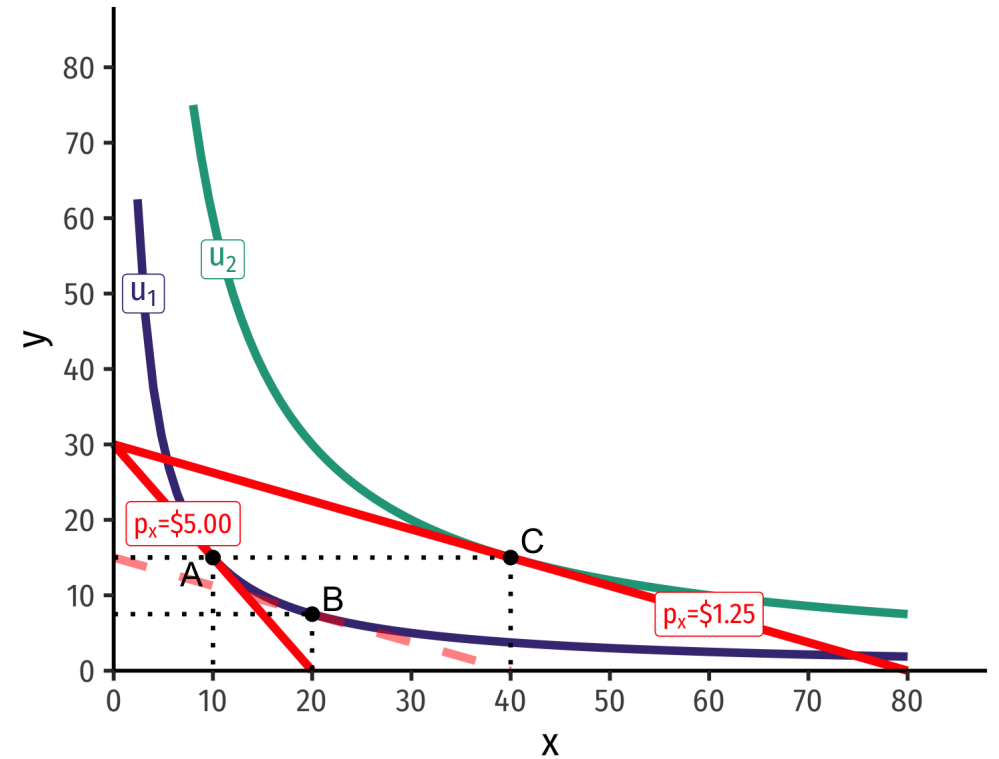


Optima with $u(x, y) = x^{0.5}y^{0.5}$, $m = 100$, $p_y = 3.33$

Real Income and Substitution Effects, Graphically III

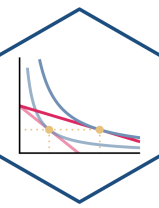


- **(Real) income effect:** change in consumption due to the **change in purchasing power** from the change in price

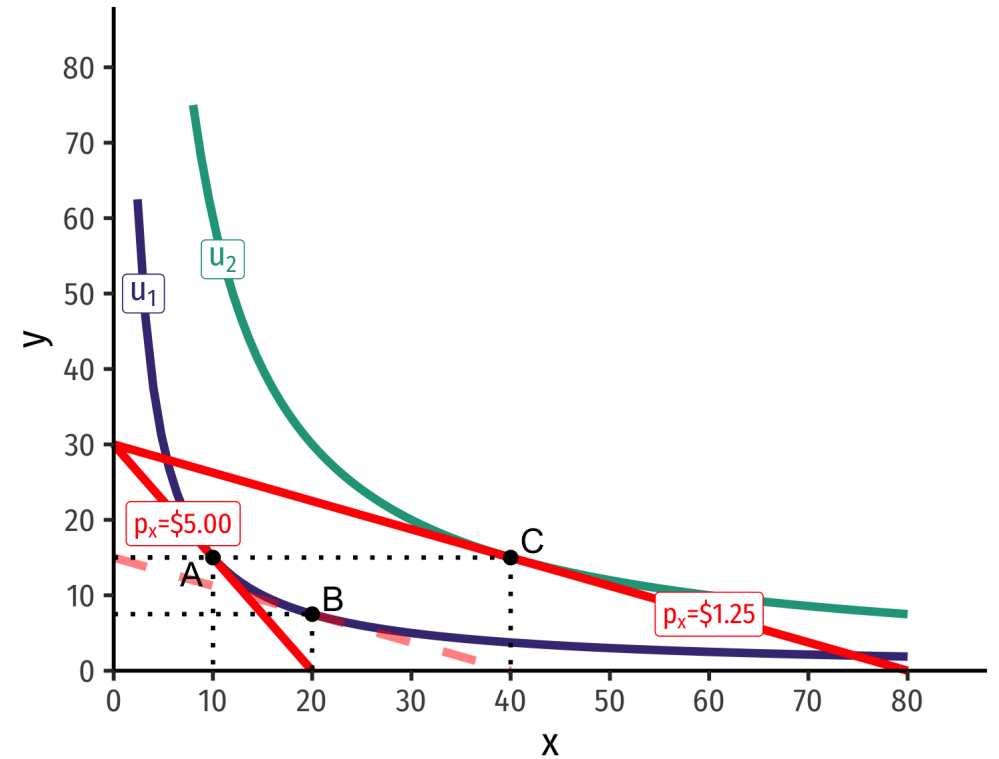


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Real Income and Substitution Effects, Graphically III

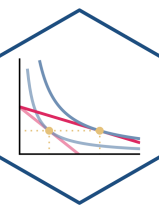


- **(Real) income effect:** change in consumption due to the **change in purchasing power** from the change in price
- $B \rightarrow C$ to new budget constraint (can buy more of x and/or y)

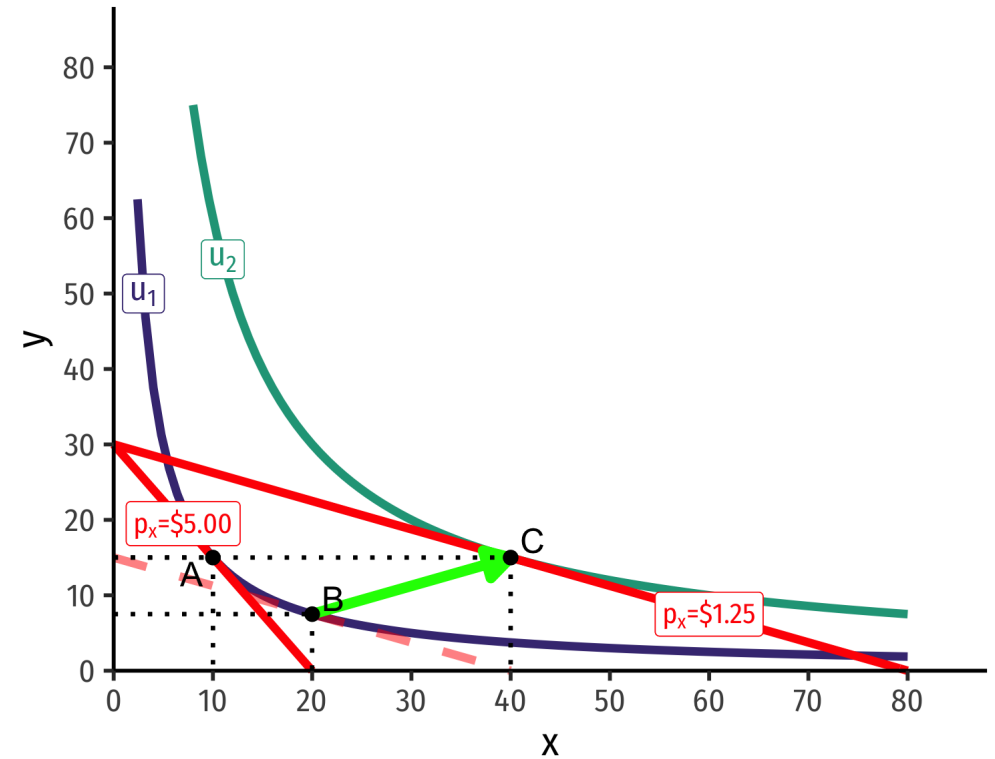


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Real Income and Substitution Effects, Graphically III



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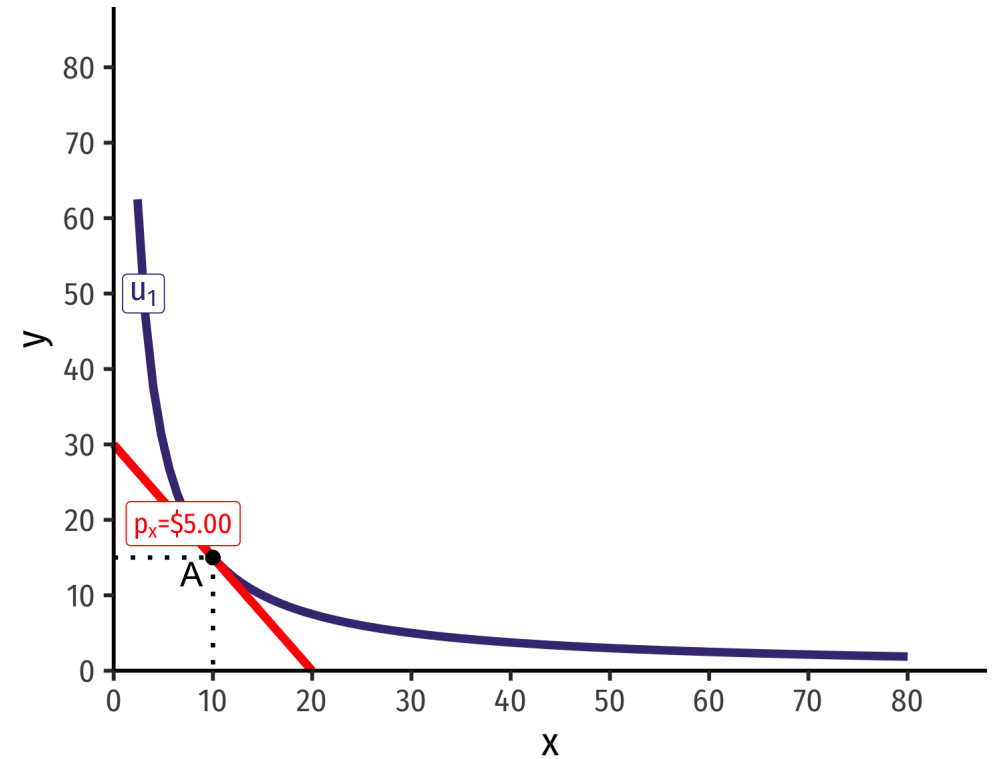


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Real Income and Substitution Effects, Graphically IV



- Original optimal consumption (A)

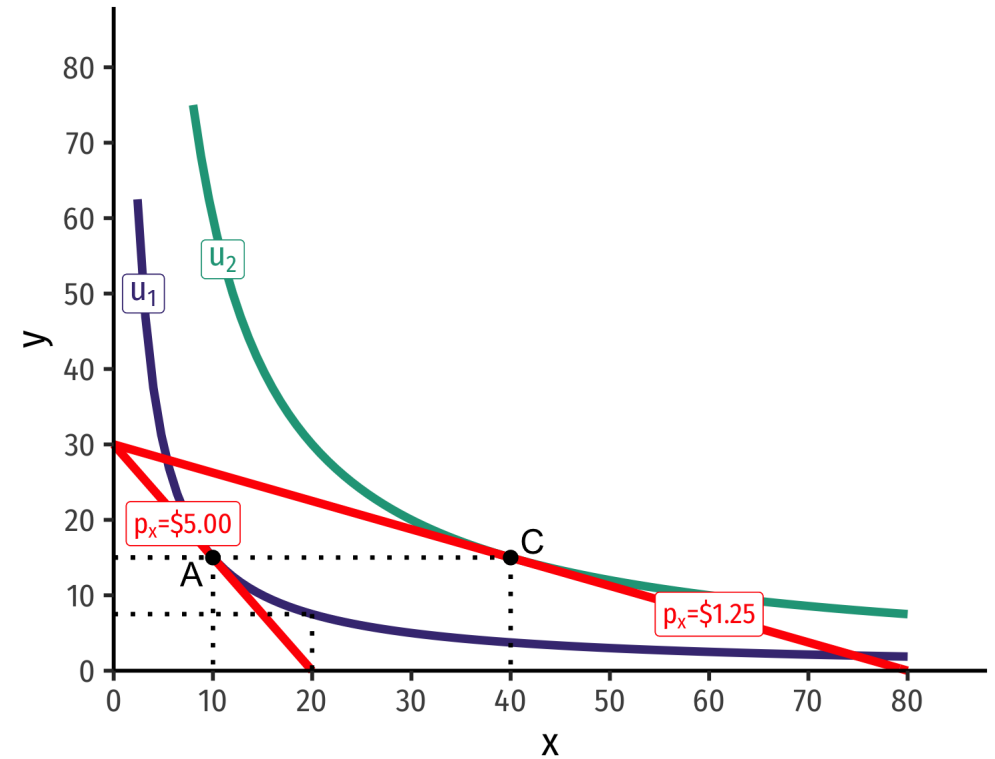


Optima with $u(x, y) = x^{0.5}y^{0.5}$, $m = 100$, $p_y = 3.33$

Real Income and Substitution Effects, Graphically IV



- Original optimal consumption (*A*)
- Price of x falls, new optimal consumption at (*C*)

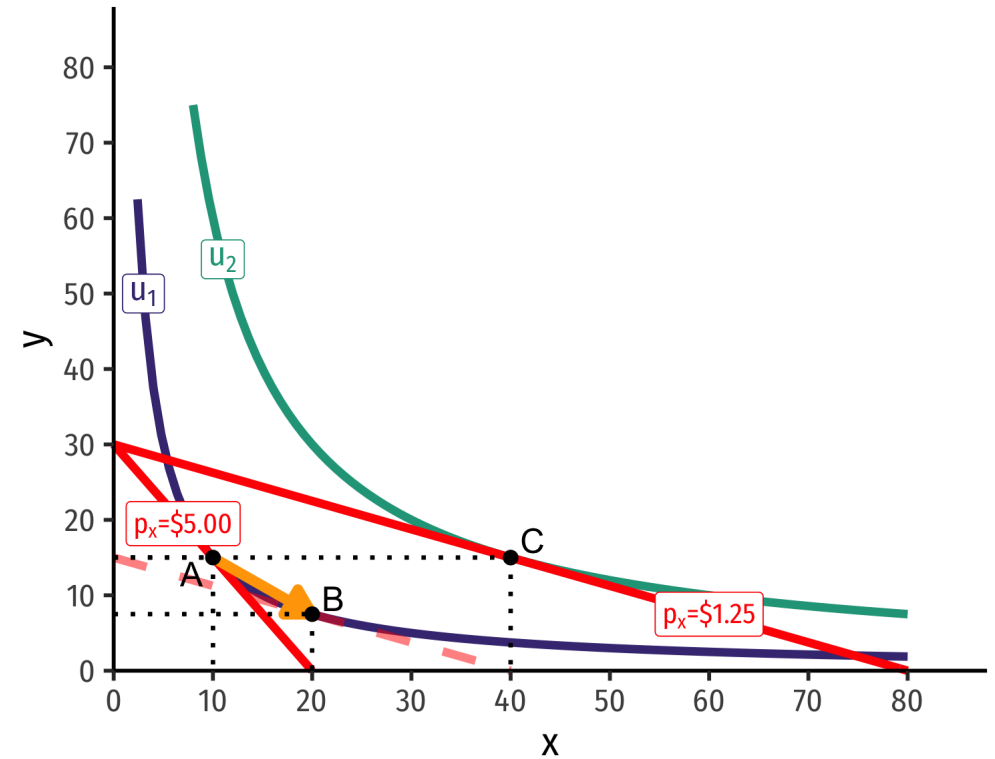


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Real Income and Substitution Effects, Graphically IV

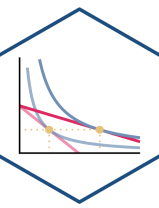


- Original optimal consumption (A)
- Price of x falls, new optimal consumption at (C)
- **Substitution effect:** $A \rightarrow B$ on same I.C. (\uparrow cheaper x and \downarrow y)

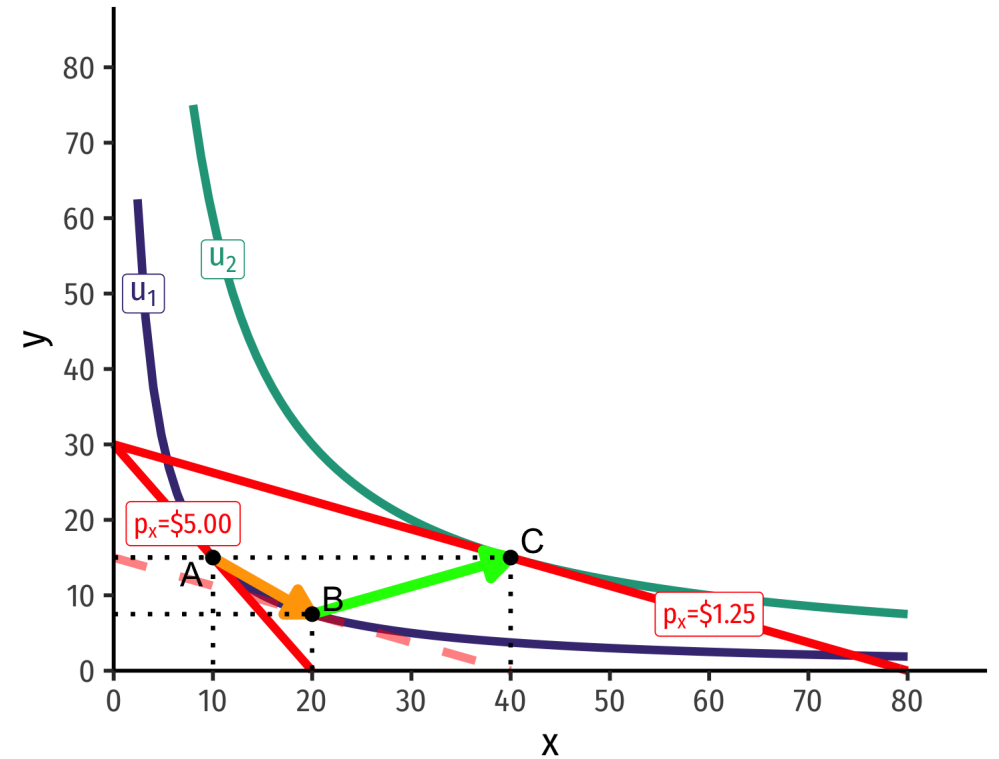


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Real Income and Substitution Effects, Graphically IV

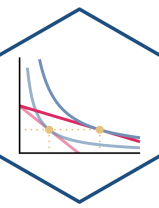


- Original optimal consumption (A)
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- **(Real) income effect:** $B \rightarrow C$ to new budget constraint (can buy more of x and/or y)

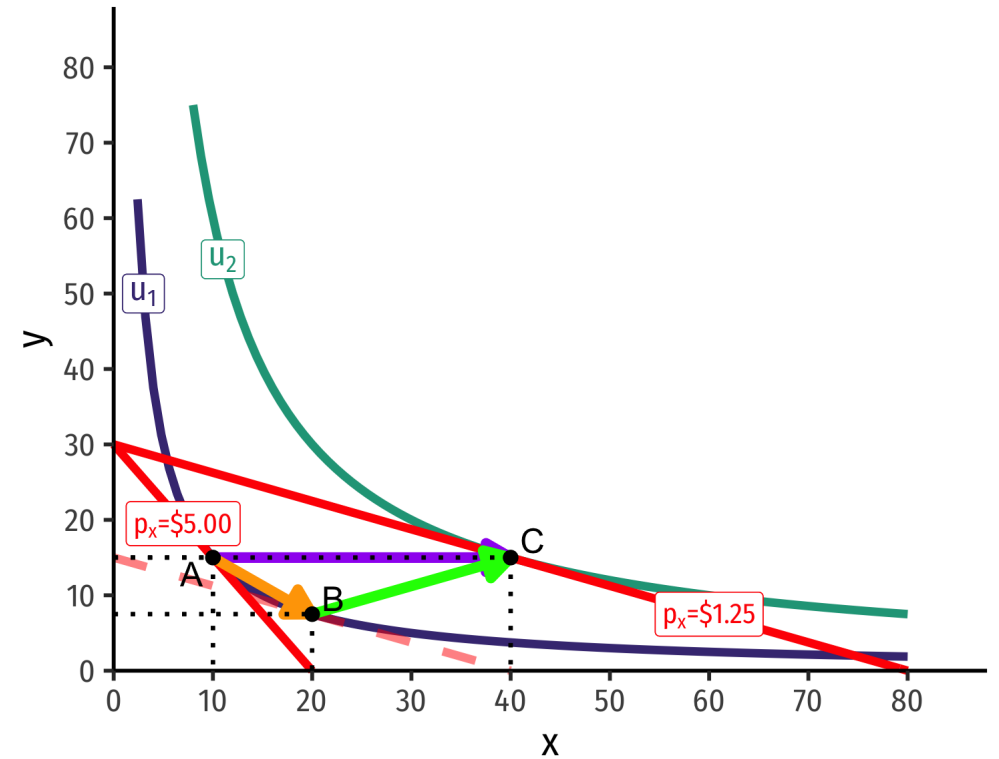


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Real Income and Substitution Effects, Graphically IV



- Original optimal consumption (A)
- Price of x falls, new optimal consumption at (C)
- **Substitution effect:** $A \rightarrow B$ on same I.C. (\uparrow cheaper x and \downarrow y)
- **(Real) income effect:** $B \rightarrow C$ to new budget constraint (can buy more of x and/or y)
- **(Total) price effect:** $A \rightarrow C$

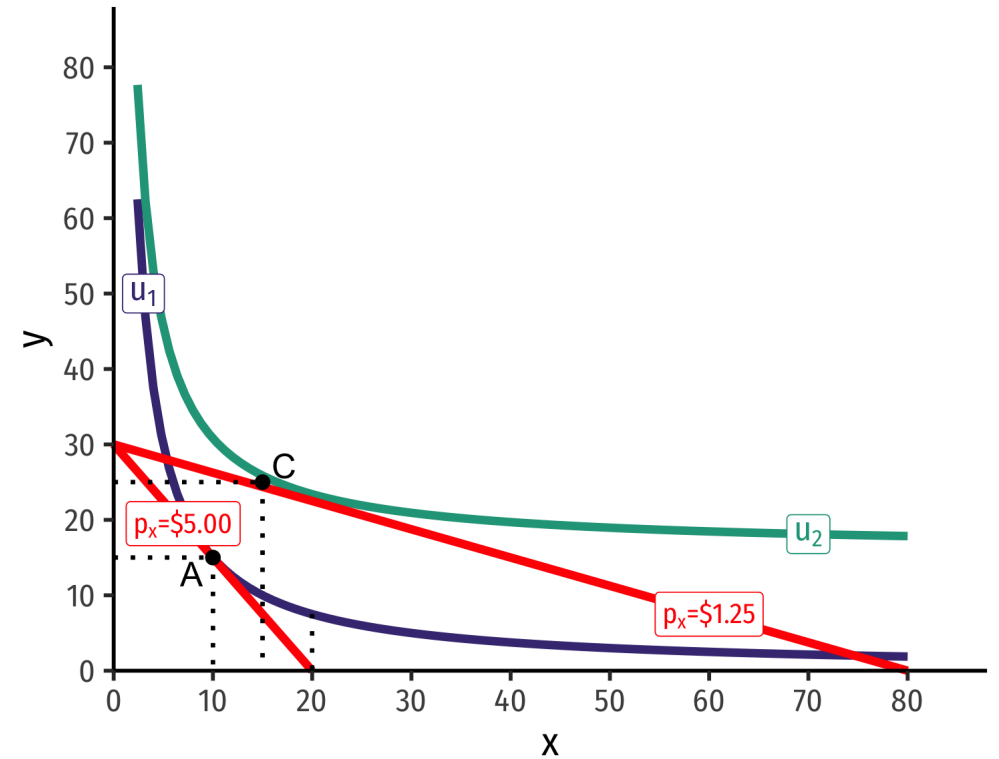


Optima with $u(x, y) = x^{0.5}y^{0.5}$, $m = 100$, $p_y = 3.33$

Real Income and Substitution Effects: Inferior Good



- What about an **inferior** good (Ramen)?

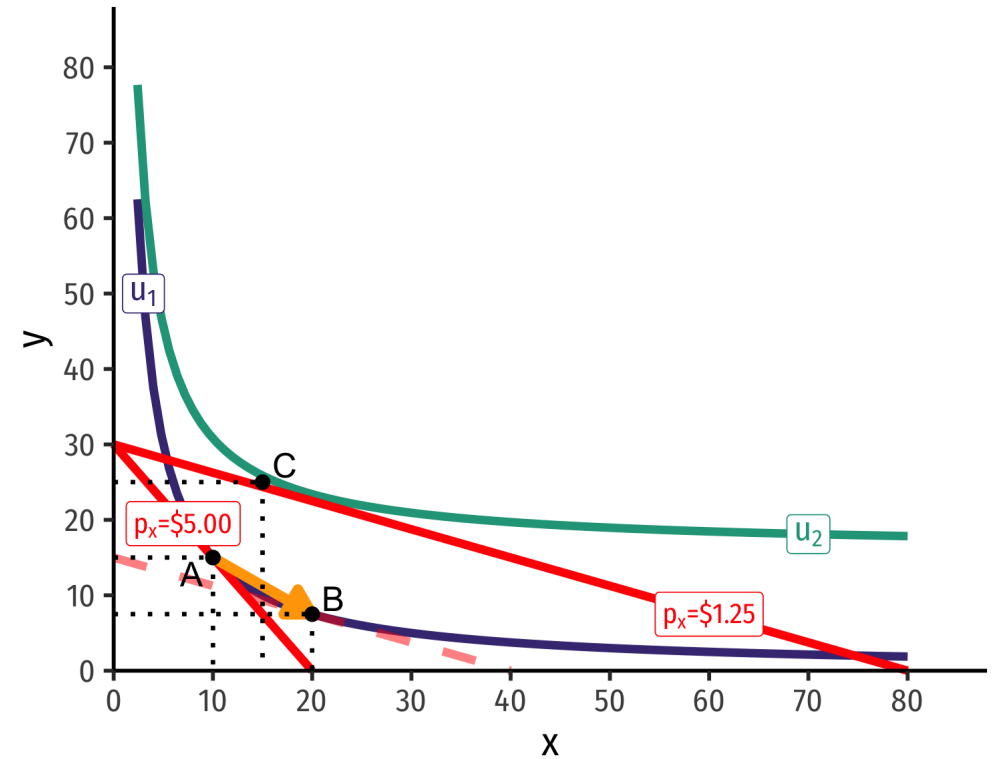


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Real Income and Substitution Effects: Inferior Good



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(\uparrow cheaper x and \downarrow y)

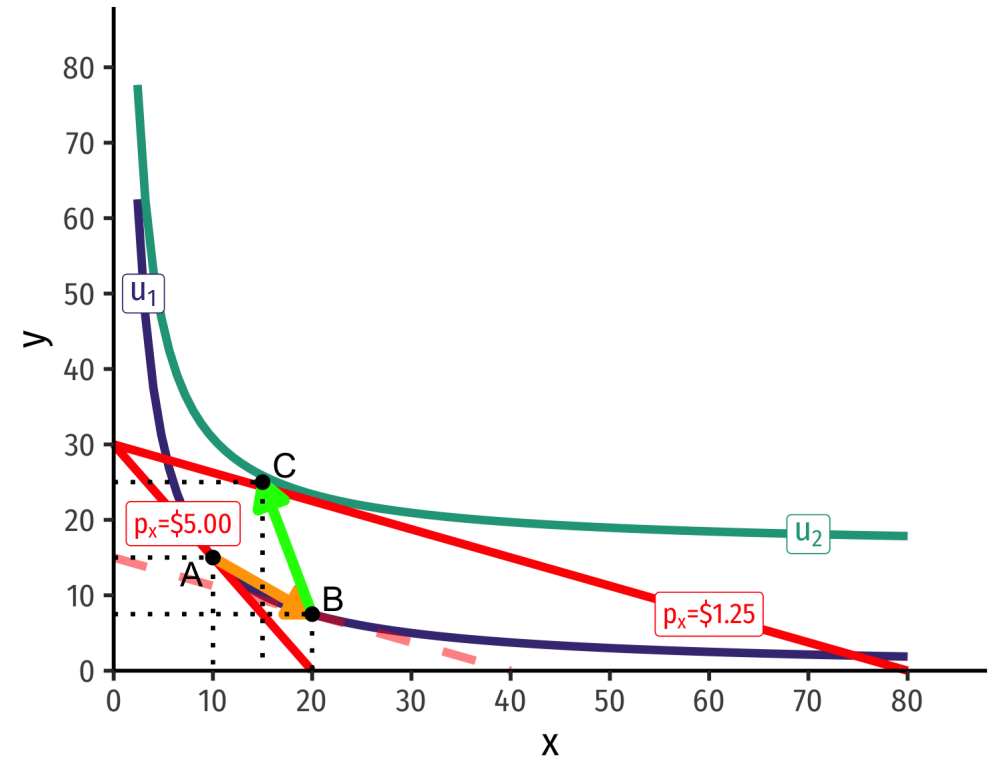


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Real Income and Substitution Effects: Inferior Good



- What about an **inferior** good (Ramen)?
- **Substitution effect:** $A \rightarrow B$ on same I.C.
(\uparrow cheaper x and \downarrow y)
- **(Real) income effect:** $B \rightarrow C$ to new budget constraint (can buy more of x and/or y) **is negative**

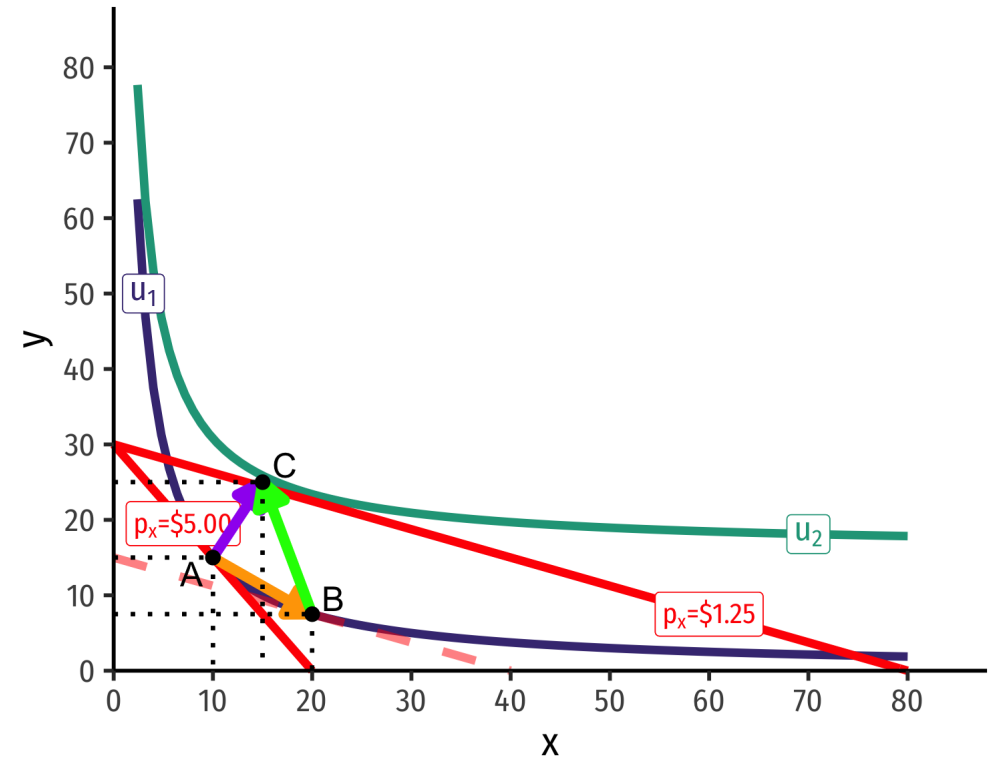


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Real Income and Substitution Effects: Inferior Good

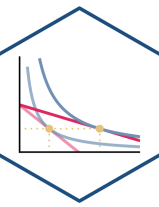


- What about an **inferior** good (Ramen)?
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- **(Total) price effect:** $A \rightarrow C$

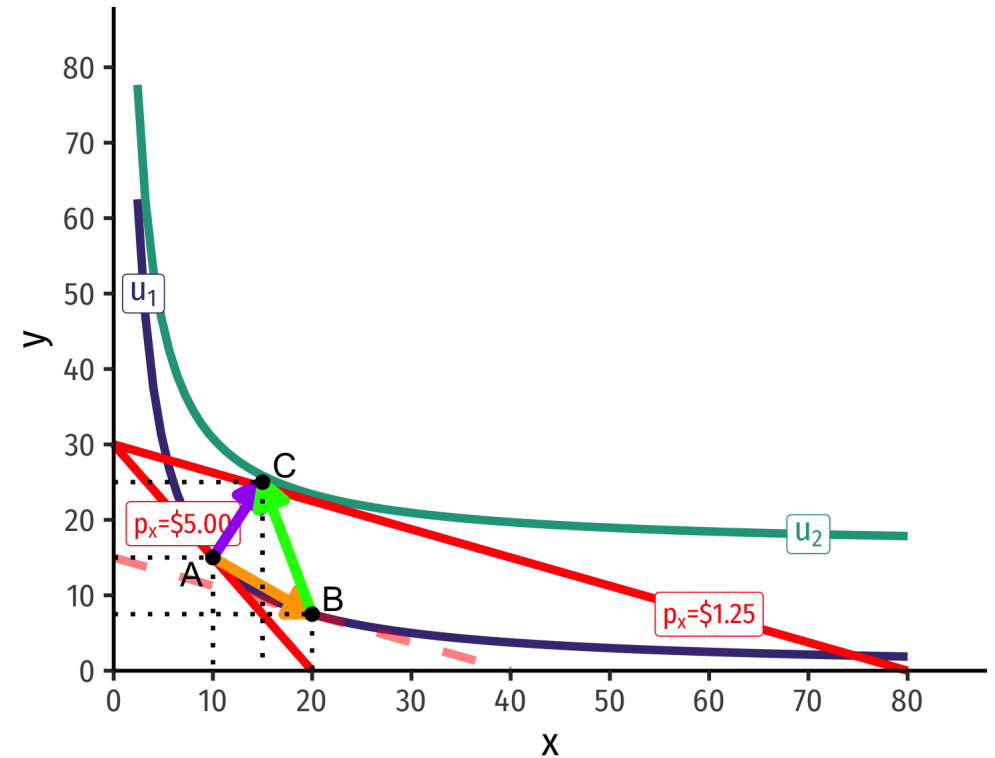


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Real Income and Substitution Effects: Inferior Good

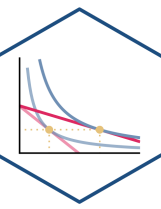


- What about an **inferior** good (Ramen)?
- **Substitution effect:** $A \rightarrow B$ on same I.C.
(\uparrow cheaper x and \downarrow y)
- **(Real) income effect:** $B \rightarrow C$ to new budget constraint (can buy more of x and/or y) **is negative**
- **(Total) price effect:** $A \rightarrow C$
- Price effect is *still* an $\uparrow x$ from a $\downarrow p_x$!
 - Person would just prefer to spend more new purchasing power on other goods



Optima with $u(x, y) = x^{0.5}y^{0.5}$, $m = 100$, $p_y = 3.33$

Violating the Law of Demand



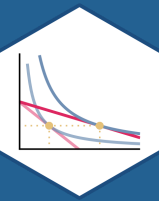
Example: What would it take to violate the law of demand?

Recap: Real Income and Substitution Effects



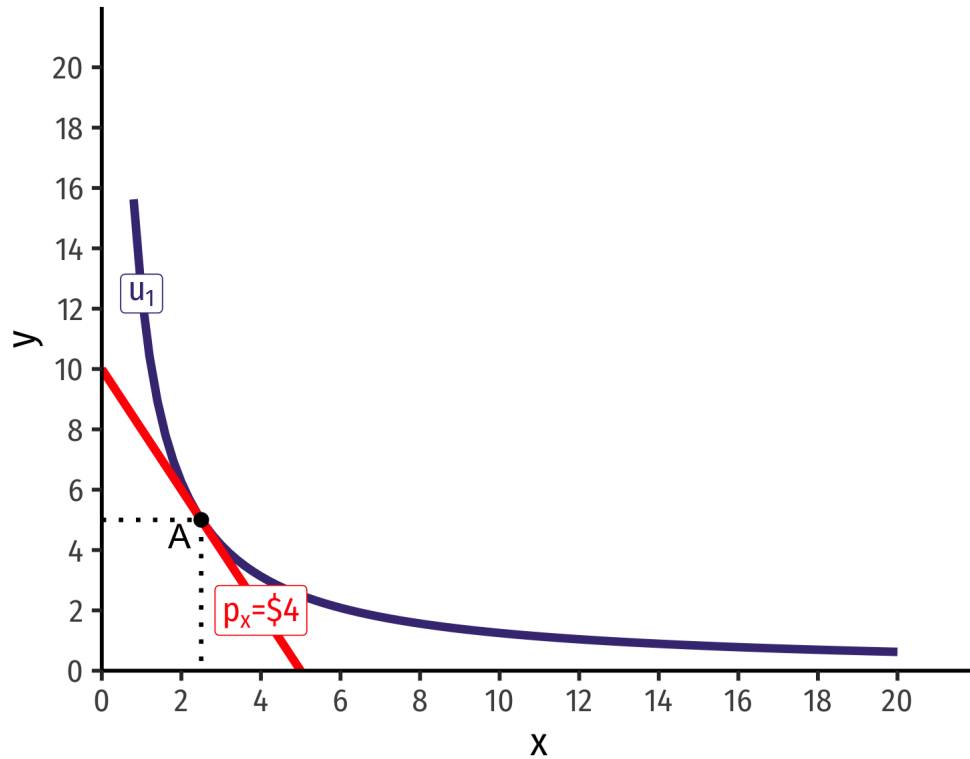
$$\text{Price Effect} = \text{Real income effect} + \text{Substitution Effect}$$

- **Substitution effect**: is always in the direction of the cheaper good
- **Real Income effect**: can be positive (normal) or negative (inferior)
- **Law of Demand**/Demand curves slope downwards (**Price effect**) mostly because of the substitution effect
 - Even (inferior) goods with negative real income effects overpowered by substitution effect
- Exception in the theoretical **Giffen good**: negative R.I.E. $>$ S.E.
 - An upward sloping demand curve!

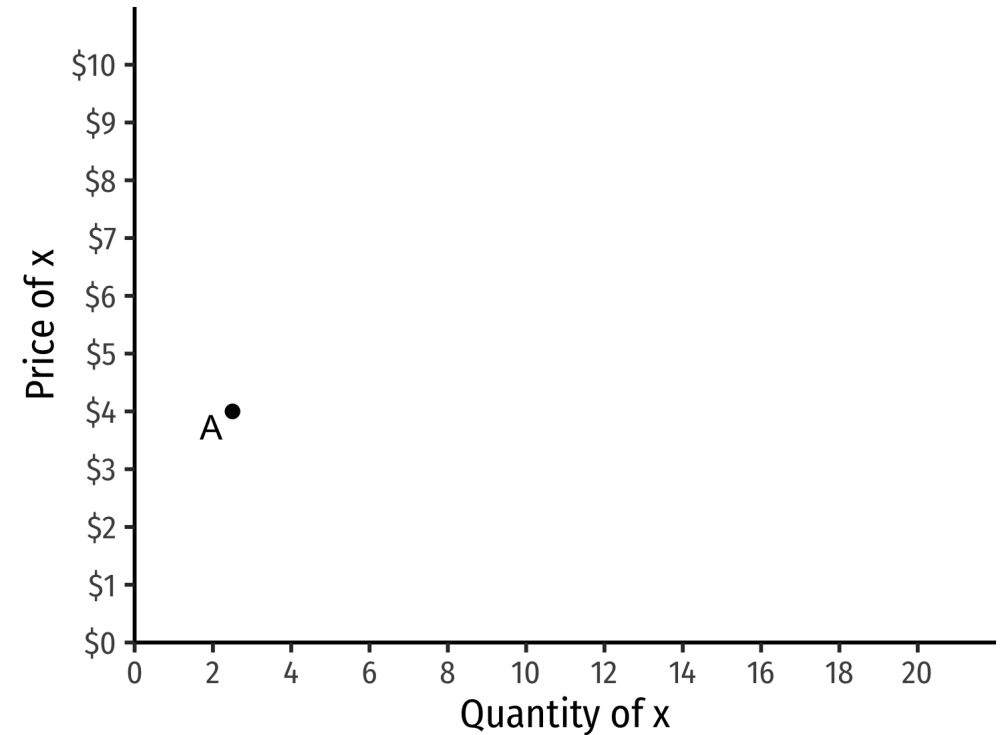


From Optimal Consumption Points to Demand

Deriving a Demand Curve Graphically



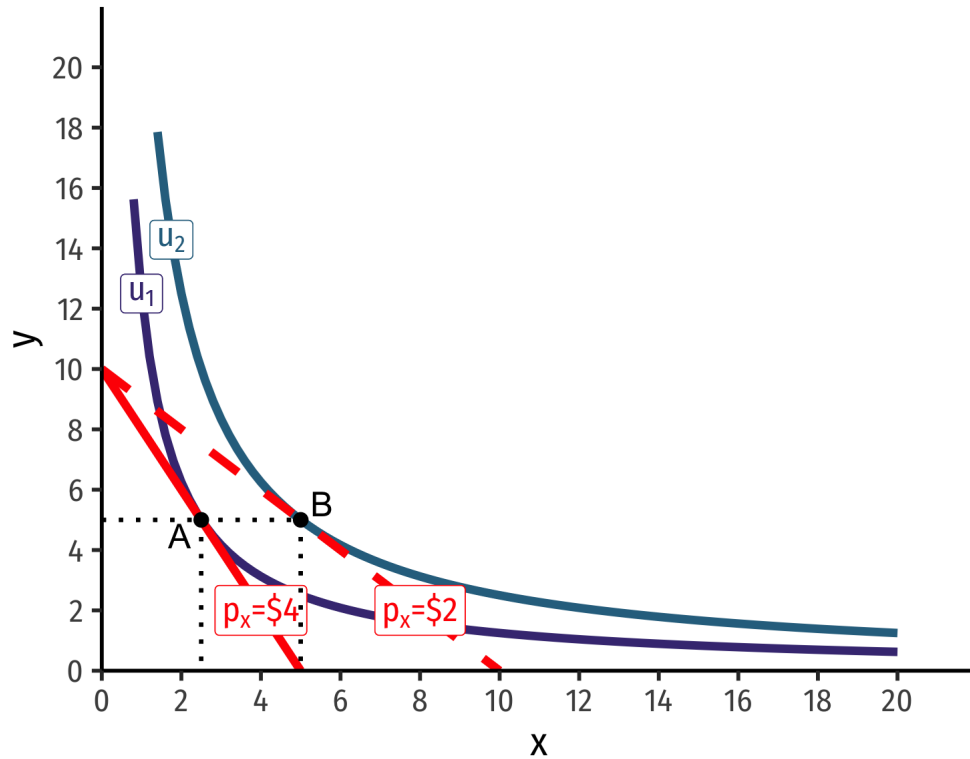
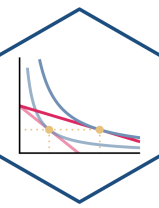
Optima with $u(x, y) = x^{0.5}y^{0.5}$, $m = 20$, $p_y = 2$



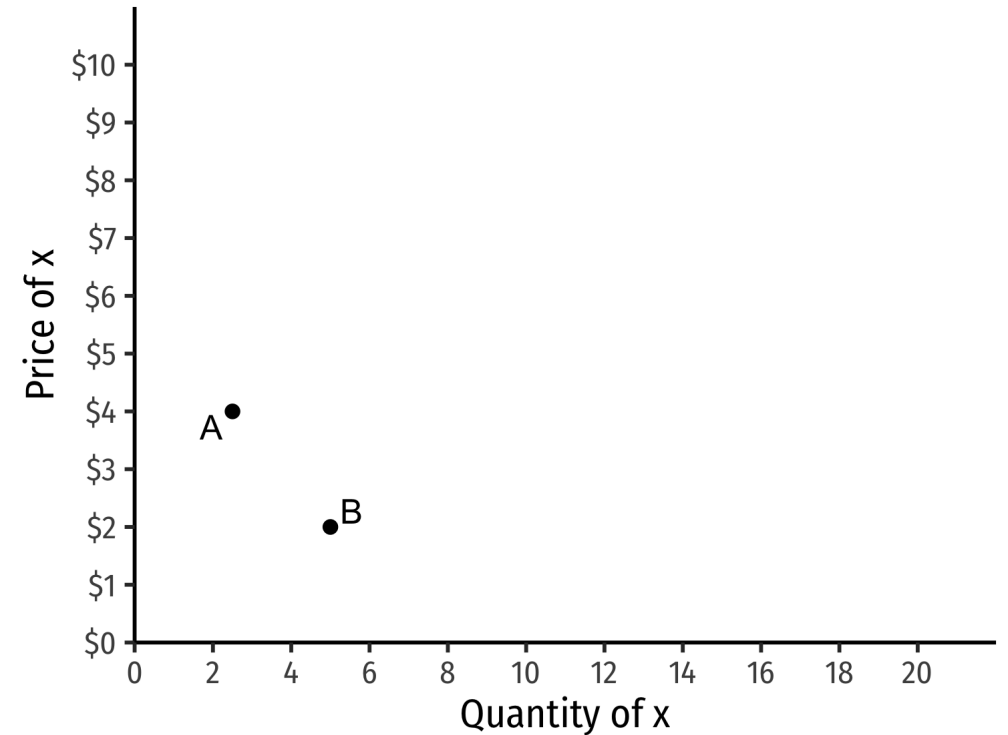
Demand function: $\frac{m}{2p}$; Inverse Demand function: $p = \frac{m}{2q}$

- Demand curve for x relates optimal consumption of x ("quantity") as price of x changes
- At $p_x = 4$, consumer buys 2 x

Deriving a Demand Curve Graphically



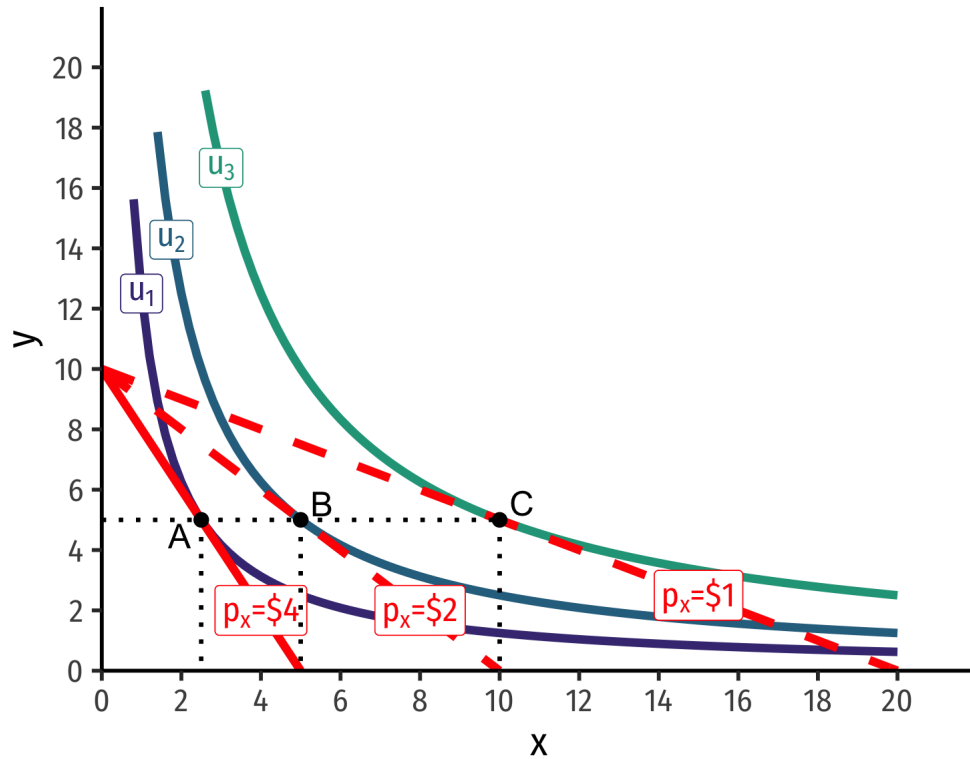
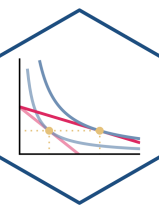
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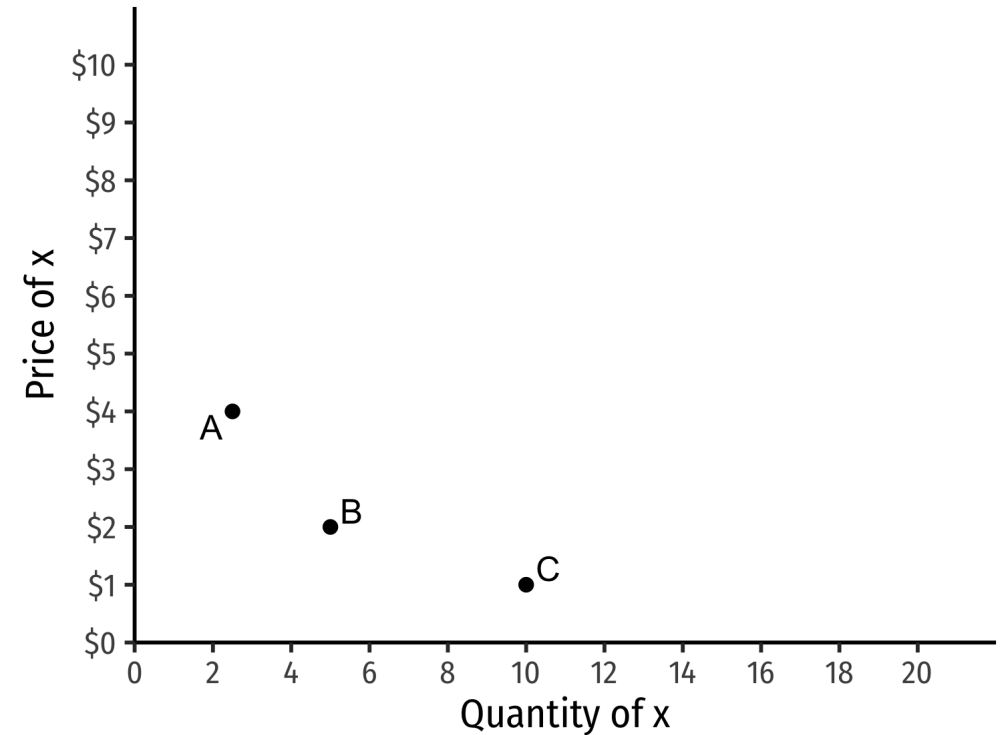
Demand function: $\frac{m}{2p}$; Inverse Demand function: $p = \frac{m}{2q}$

- Demand curve for x relates optimal consumption of x ("quantity") as price of x changes
- At $p_x = 4$, consumer buys 2 x ; at $p_x = 2$, consumer buys 5 x

Deriving a Demand Curve Graphically



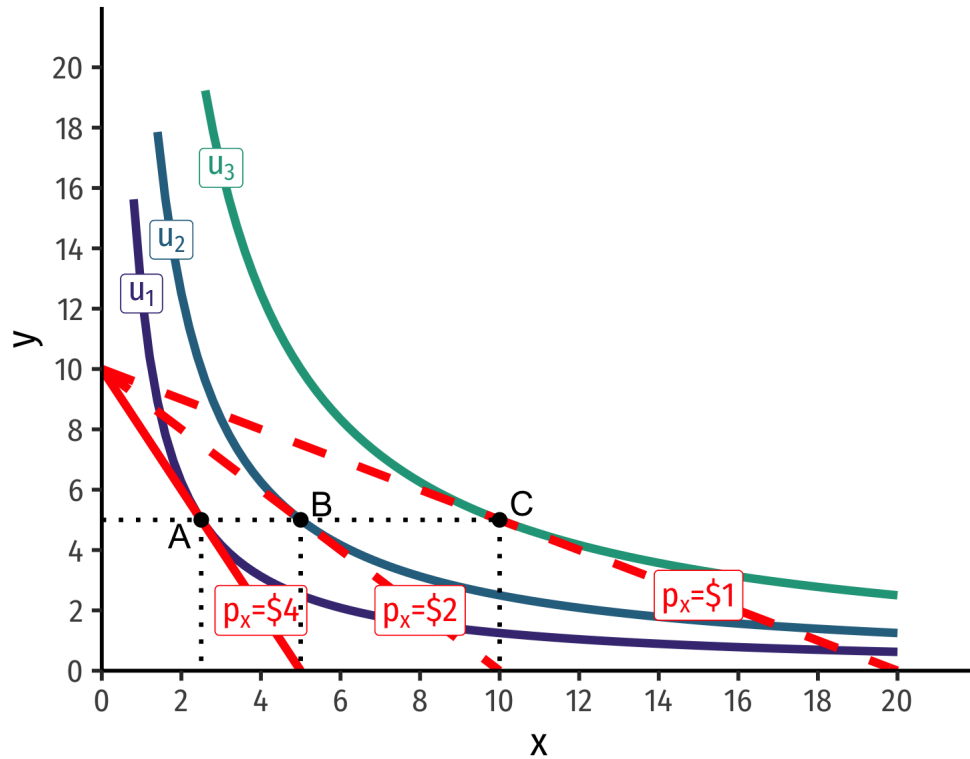
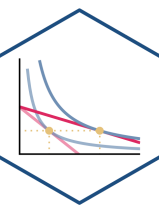
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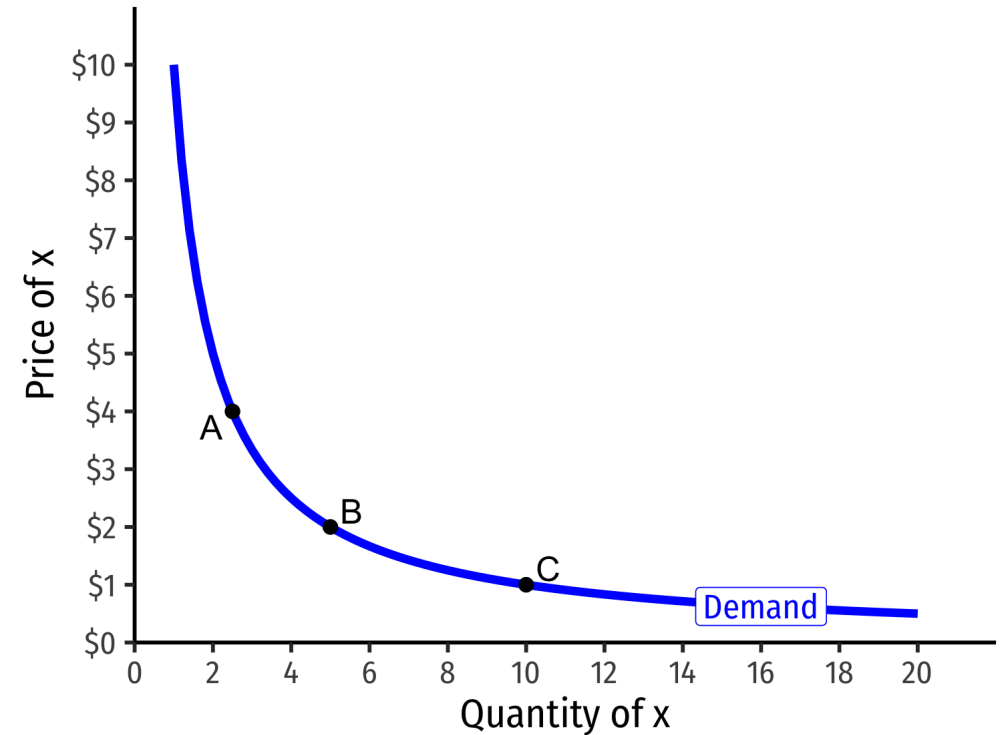
Demand function: $\frac{m}{2p}$; Inverse Demand function: $p = \frac{m}{2q}$

- Demand curve for x relates optimal consumption of x ("quantity") as price of x changes
- At $p_x = 4$, consumer buys 2 x ; at $p_x = 2$, consumer buys 5 x ; at $p_x = 1$, consumer buys 10 x

Deriving a Demand Curve Graphically



Optima with $u(x, y) = x^{0.5}y^{0.5}$, $m = 20$, $p_y = 2$



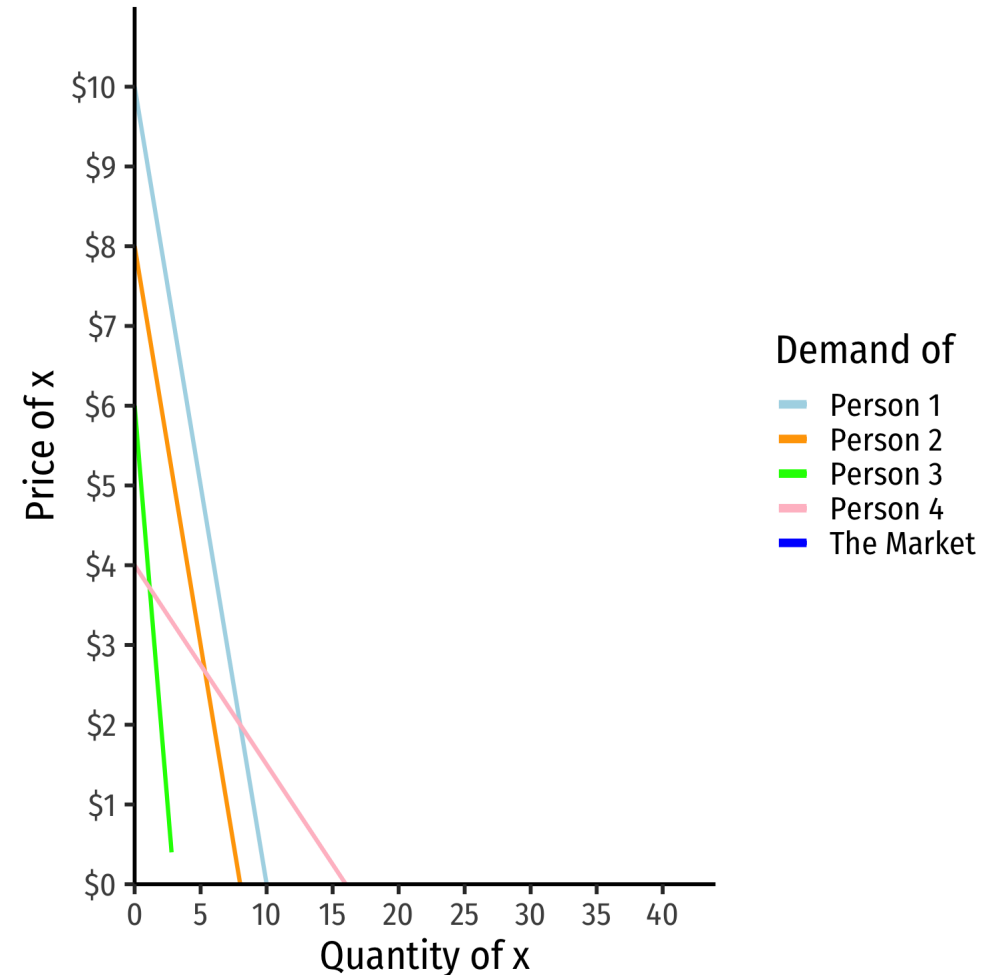
Demand function: $\frac{m}{2p}$; Inverse Demand function: $p = \frac{m}{2q}$

- Demand curve for x relates optimal consumption of x ("quantity") as price of x changes
- At $p_x = 4$, consumer buys 2 x ; at $p_x = 2$, consumer buys 5 x ; at $p_x = 1$, consumer buys 10 x

From Individual Demand to Market Demand



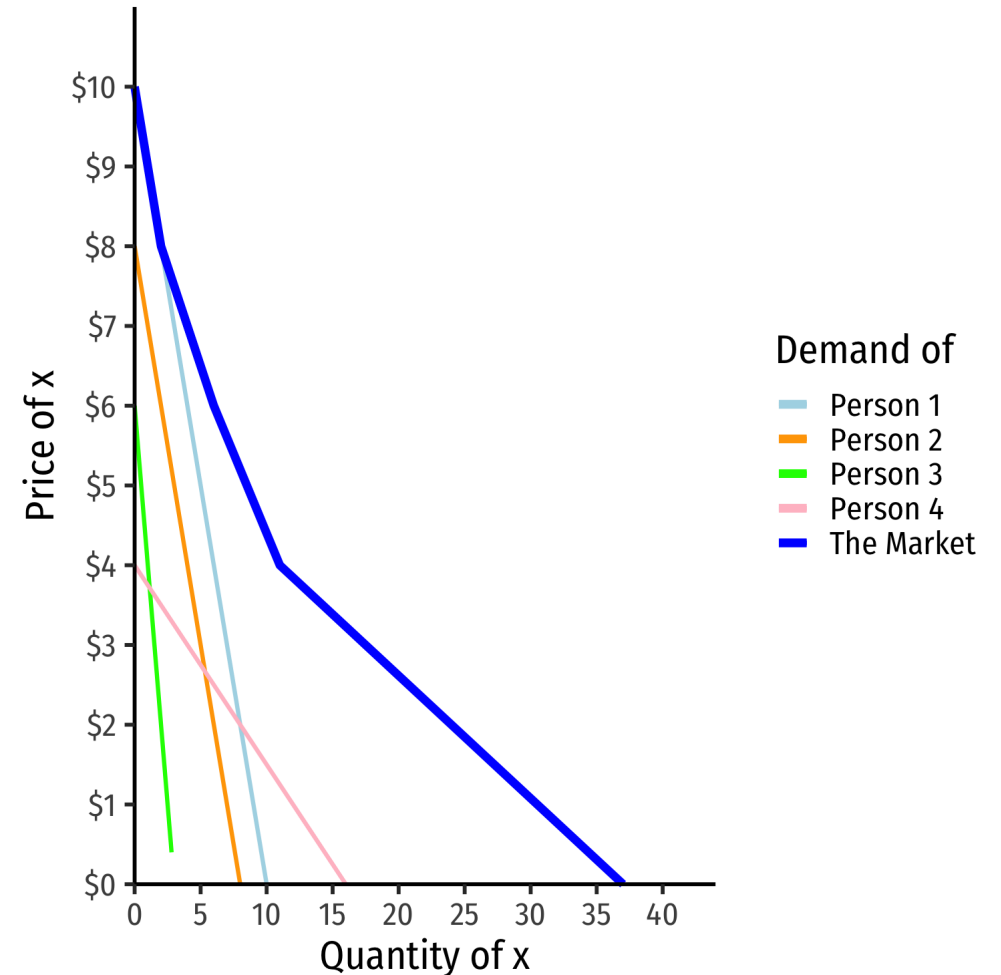
- Note so far we have been talking about *an individual person's* demand
- In principles, you learned about the entire **market demand**



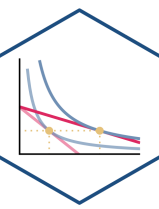
From Individual Demand to Market Demand



- Note so far we have been talking about *an individual person's* demand
- In principles, you learned about the entire **market demand**
- This is simply the sum of all individuals' demands



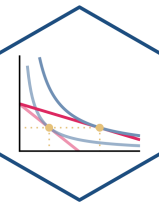
Demand Schedule (For Individual Or Market)



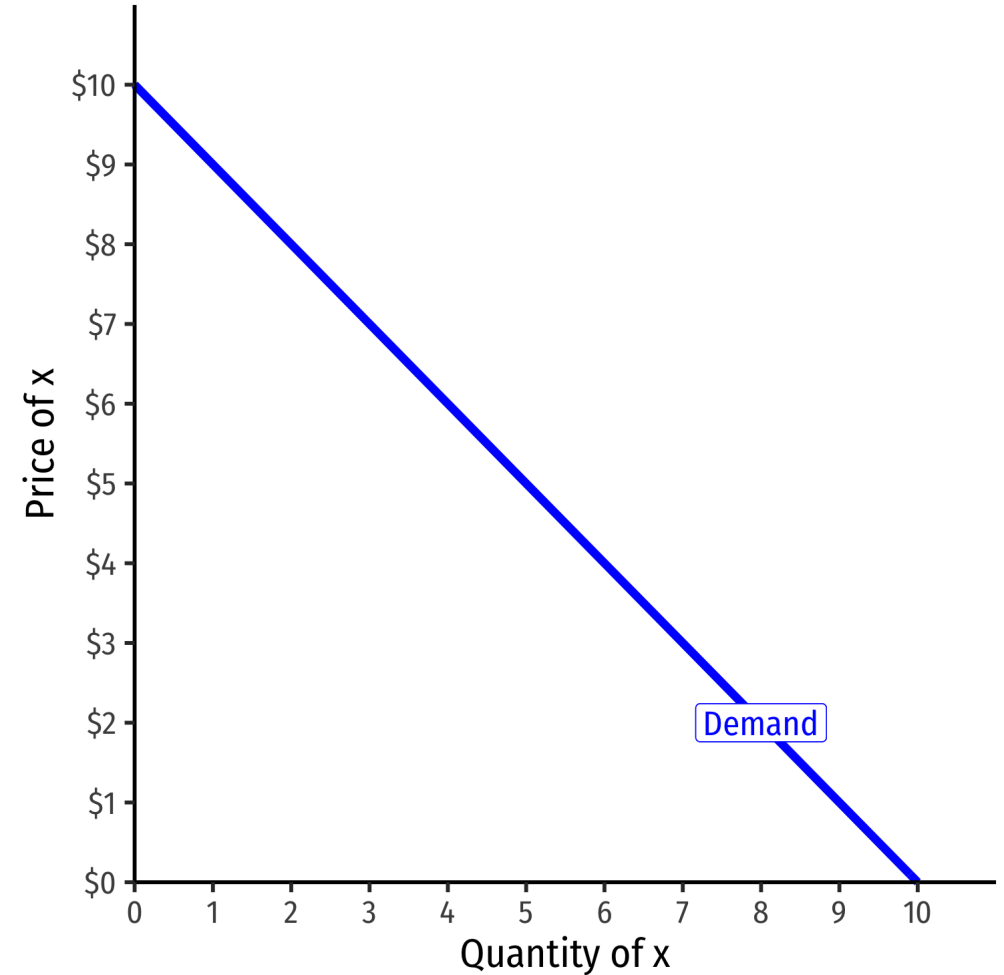
- **Demand schedule** expresses the quantity of good a person(s) would be willing to buy (q_D) at any given price (p_x)
 - Holding constant all other prices (p_y) and income (m)! (“**ceterus paribus**”)
- Note: **each of these is a consumer's optimum at a given price!**

price	quantity
10	0
9	1
8	2
7	3
6	4
5	5
4	6
3	7
2	8
1	9
0	10

Demand Curve



- **Demand curve** graphically represents the demand schedule
- Also measures a person's **maximum willingness to pay (WTP)** for a given quantity
- **Law of Demand (price effect)** \implies demand curves always slope downwards



Demand Function



- **Demand function** relates quantity to price

Example:

$$q = 10 - p$$

- Not graphable (wrong axes)!

Inverse Demand Function

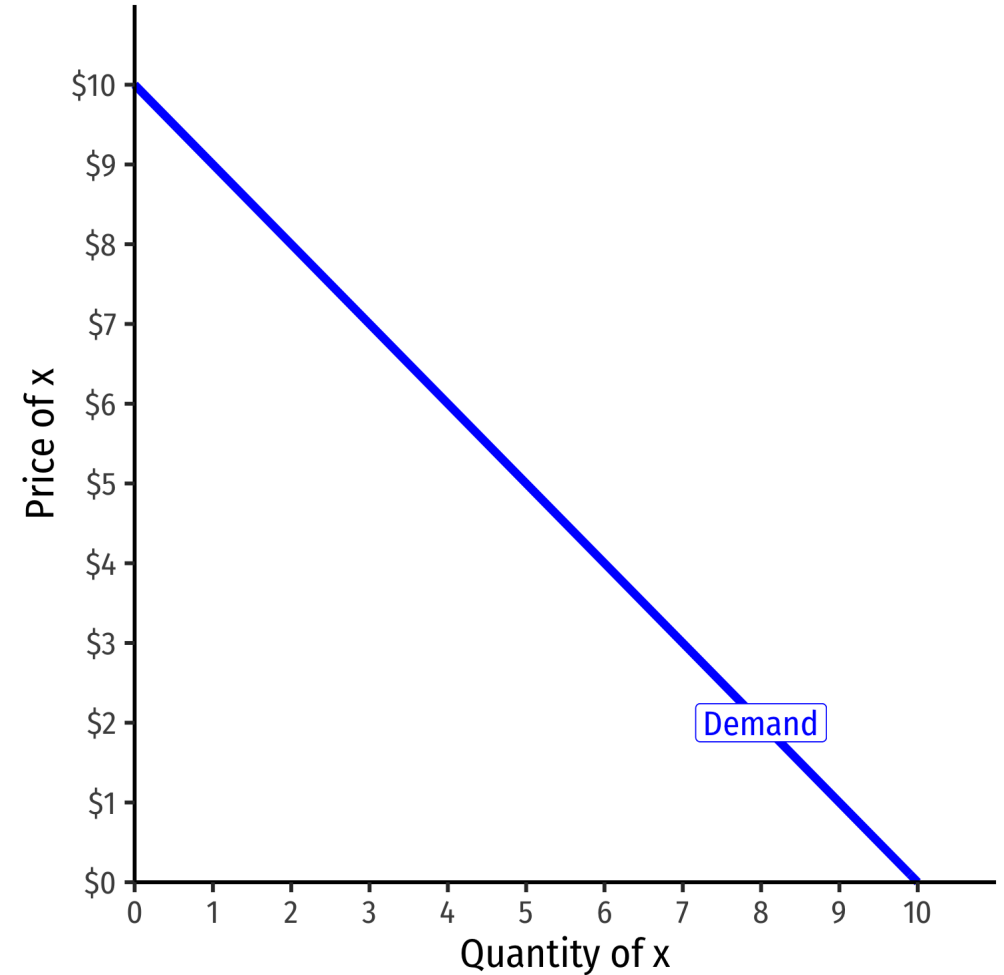


- **Inverse demand function** relates price to quantity
 - Take demand function and solve for p

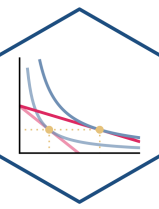
Example:

$$p = 10 - q$$

- Graphable (price on vertical axis)!



Inverse Demand Function

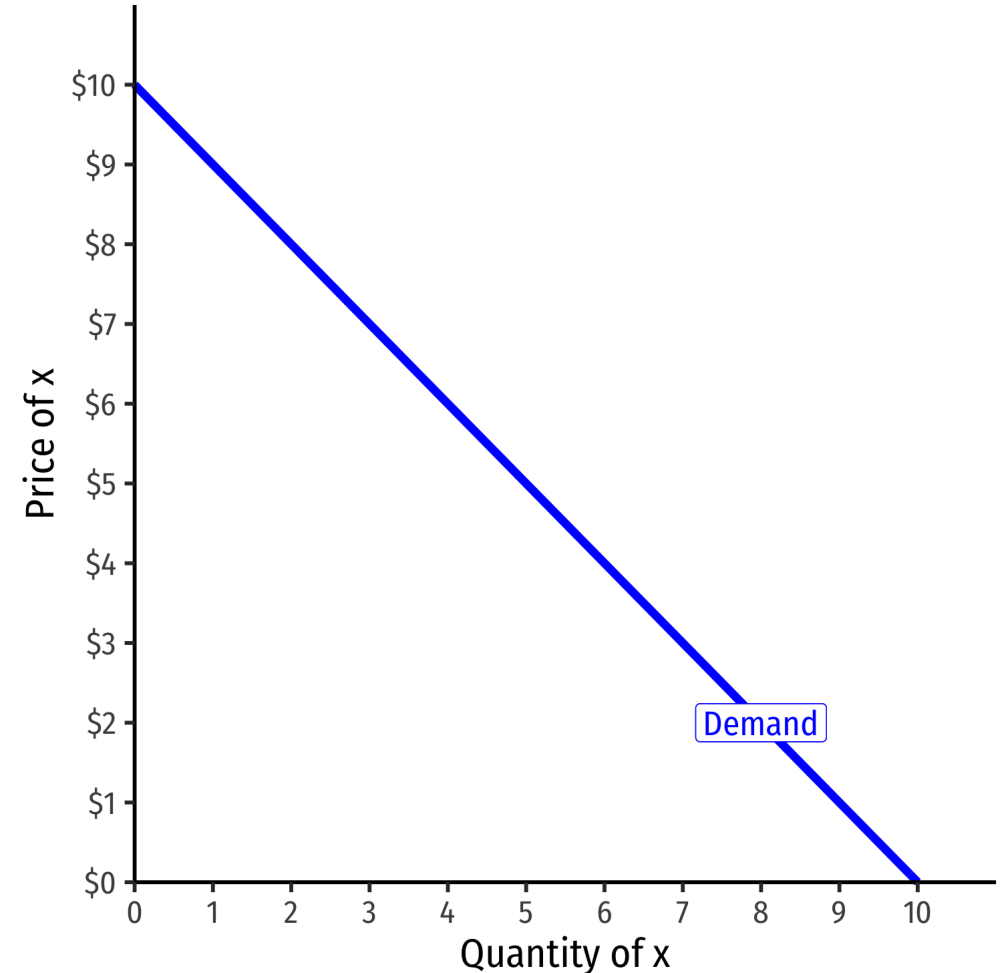


- **Inverse demand function** relates price to quantity
 - Take demand function and solve for p

Example:

$$p = 10 - q$$

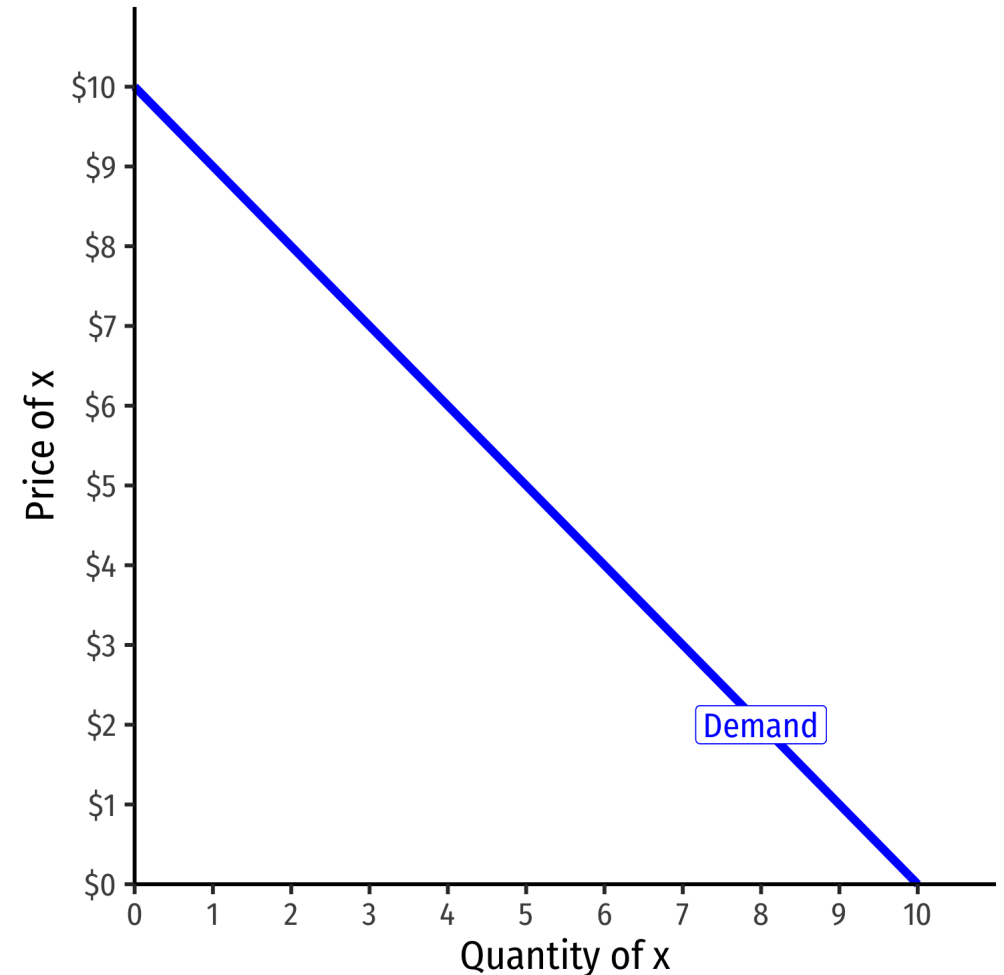
- Vertical intercept ("**Choke price**"): price where $q_D = 0$ (\$10), just high enough to discourage *any* purchases



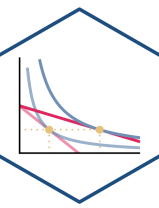
Inverse Demand Function



- Read two ways:
- Horizontally: at any given price, how many units person wants to buy
- Vertically: at any given quantity, the **maximum willingness to pay (WTP)** for that quantity
 - This way will be very useful later



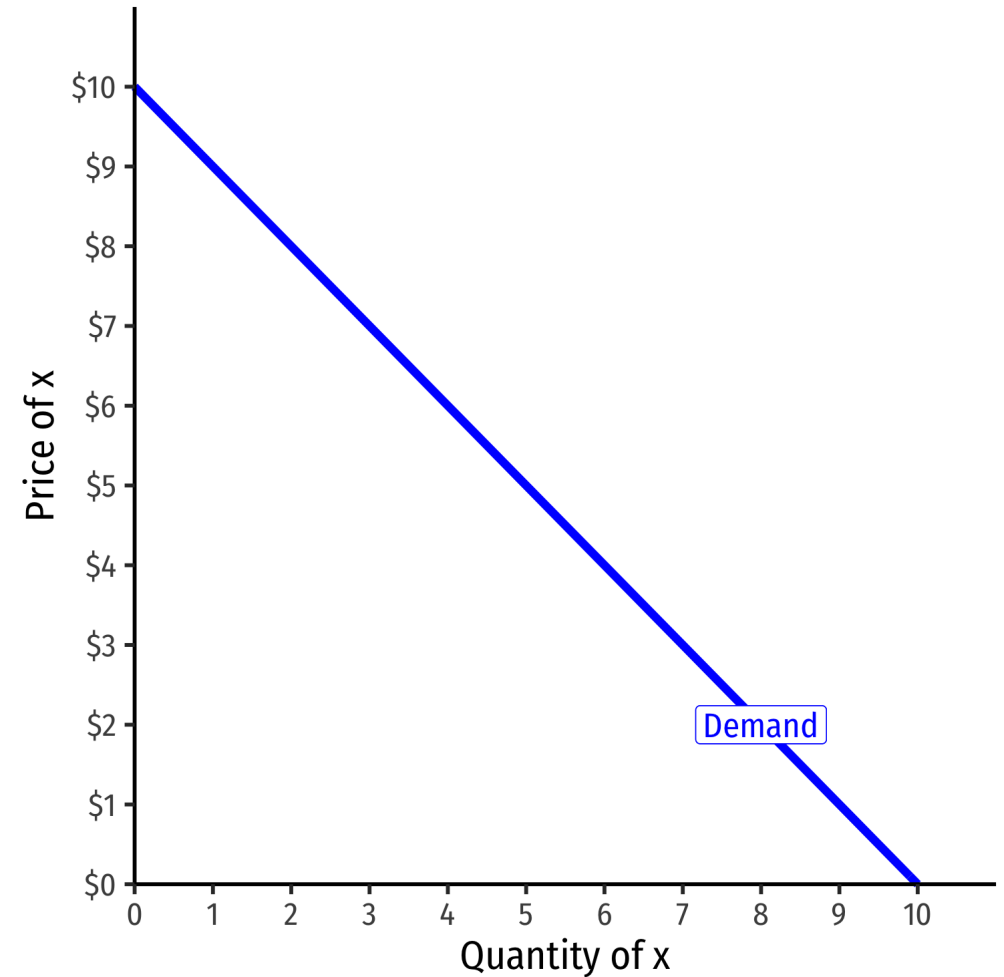
Shifts in Demand I



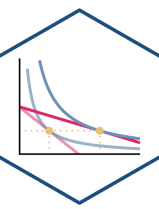
- Note a simple (inverse) demand function only relates (own) **price** and **quantity**

Example: $q = 10 - p$ or
 $p = 10 - q$

- What about all the other "**determinants of demand**" like income and other prices?



Shifts in Demand II



- A change in one of the "**determinants of demand**" will **shift** demand curve!
 1. Change in **income** m
 2. Change in **price of other goods** p_y
 3. Change in **preferences** or **expectations** about good x
- Shows up in (inverse) demand function by a **change in intercept (choke price)**!
- See my [Visualizing Demand Shifters](#)

