Consumer Theory Application IV: Exchange, Efficiency, & Equilibrium

Ryan Safner

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We have examined the case of a single individual (a consumer) making optimal choices given market prices, income, and their preferences. In one sense, an economy is just a collection of millions of these individuals all doing the same thing. But economics truly begins where at the more important stage where these individuals interact and exchange *with one another*. In particular, we want to examine how relative prices and income are *determined* in an economy (up until now we simply *assumed* them as *given* parameters). They must come from somewhere! – the interactions of individuals buying and selling with one another, responding in order to balance supply and demand. This is where the study of *equilibrium* comes from, and what the traditional Supply and Demand model captures in its simplifications.

We start with a single person who can consume Apples (a) and Bananas (b). Suppose that this person *starts* with an **endowment** of each good:

- $(\bar{a}, \bar{b})$: amount of apples and bananas person 1 has to begin with

Here we are assuming a simple **endowment economy**: apples and bananas only exist in the possession of a person. Later we will introduce a second person, who also may have apples and bananas, and the two can exchange with each other. Apples and Bananas are not produced, there are no firms. We are going to focus on the effects of exchange alone (later we will introduce firms and production of new goods).

Recall the original consumer's problem:

- **Choose:** <bundle>
- **In order to maximize:** <utility function>
- **Subject to:** <budget constraint>

The budget constraint was given by:

$$p_a a + p_b b = m$$

Now, instead of the mysterious income m, the income a person gets comes from the *value of their endowment*:

$$p_a a + p_b b = p_a \bar{a} + p_b \bar{b}$$

We can rearrange this to equivalently say that:

$$p_a(a - \bar{a}) + p_b(b - \bar{b}) = 0$$

Where

- $(a - \bar{a})$ is the person's *net demand* for apples
- $(b - \bar{b})$ is the person's *net demand* for bananas

We ordinarily talk about a person's *demand* for a good, say good a as the quantity a person desires to purchase. If the person has an endowment of that good ($\bar{a}$) that they start off with, they may still wish to purchase *more* of good a, so the *net demand* would be positive. Gross demand is how much the individual ultimately consumes, whereas net demand is how much was actually bought or sold on the market (rather than just consuming the endowment).

If net demand is negative (e.g. suppose $a - \bar{a} < 0$), the person wishes to consume less of the good than they have, then they will become a *seller* of the good. A negative net demand is the amount supplied.

- If $(a - \bar{a}) > 0$: Person will buy apples
- If $(a - \bar{a}) < 0$: Person will sell apples

Remember, on the whole, in an exchange economy, before consumption there is always the same amount of goods–goods do not appear or disappear, they only change hands depending on if one wishes to buy or sell.

1 Simple Graph of 1 Person

We can examine the consumer's problem graphically, as usual, with some very slight modifications. The person's budget constraint must always go through the person's pre-existing endowment of apples and bananas $(\bar{a}, \bar{b})$. In other words, by definition, the endowment must always be affordable.

Suppose a person begins with their endowment of 1 apple and 9 bananas at point E. Suppose their utility function is:

$$u(a, b) = ab$$

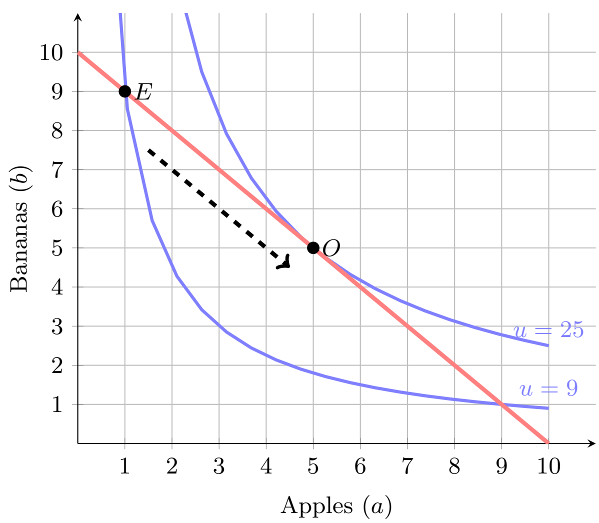
Then they earn a utility of 9. If there are trading opportunities in the market (represented by the budget line), then they can buy and/or sell apples and bananas to earn higher utility. In this particular graph, with a relative price of $\frac{p_a}{p_b} = \frac{1}{1} = 1$ and an income of \$10, the optimum of this person is to consume 5 apples and 5 bananas at point O. (I leave it as an exercise for you to prove, in the usual way, that this is the optimum bundle to consume).

What we are examining now is that the person's net demands:

$$\begin{aligned}
\text{Net Demand}_{apples} &= (a - w_a)\\
\text{Net Demand}_{apples} &= 5 - 1\\
\text{Net Demand}_{apples} &= 4
\end{aligned}$$

$$\begin{aligned}
\text{Net Demand}_{bananas} &= (b - w_b)\\
\text{Net Demand}_{bananas} &= 9 - 5\\
\text{Net Demand}_{bananas} &= -4
\end{aligned}$$

That is, this person begins with 1 apple and 9 bananas, and would like to optimally consume 5 apples and 5 bananas, meaning this person will need to buy 4 apples and sell 4 bananas in the market. This person moves along the trading line (budget line) in the graph, from point E to point O, exchanging apples and bananas at the market-determined price of 1:1. At the optimum, the person earns a higher utility of 25.



10
9
8
7
6
5
4
3
2
1
Bananas (b)
E
O
$u = 25$
$u = 9$
1 2 3 4 5 6 7 8 9 10
Apples (a)

2 Trade with 2 People & General Equilibrium

Now, we have always just been looking at one side of the market, the buyer. In our simple endowment economy of two people and two goods, there must always be someone on the other side of an exchange, so we introduce person 2.

In addition to our person 1 depicted above, let's assume that person 2 begins with an endowment of 9 apples and 1 banana, and for simplicity, has a preferences according to the (identical) utility function:

$$u(a, b) = ab$$

With their endowment, they earn a utility of 9. This is depicted as point E^2. (We will use the superscript ^{1} to denote person 1 and ^{2} to denote person 2).

Let's examine now person 2's net demands:

$$\begin{aligned}
\text{Net Demand}_{apples} &= (a^2 - \bar{a}^2) \\
\text{Net Demand}_{apples} &= 5 - 9 \\
\text{Net Demand}_{apples} &= 4
\end{aligned}$$

$$\begin{aligned}
\text{Net Demand}_{bananas} &= (b^2 - \bar{b}^2) \\
\text{Net Demand}_{bananas} &= 1 - 5 \\
\text{Net Demand}_{bananas} &= 4
\end{aligned}$$

That is, this person begins with 9 apples and 1 banana, and would like to optimally consume 5 apples and 5 bananas, meaning this person will need to sell 4 apples and buy 4 bananas in the market. This person moves along the trading line (budget line) in the graph, from point E^2 to point O^2, exchanging apples and bananas at the market-determined price of 1:1. At the optimum, the person earns a higher utility of 25.

Note that person 2 must face the *same* budget line as person 1, because the budget line represents the market tradeoff, and therefore the exchange opportunities in the market. In this case, the "market" is literally just person 1 and person 2 exchanging!

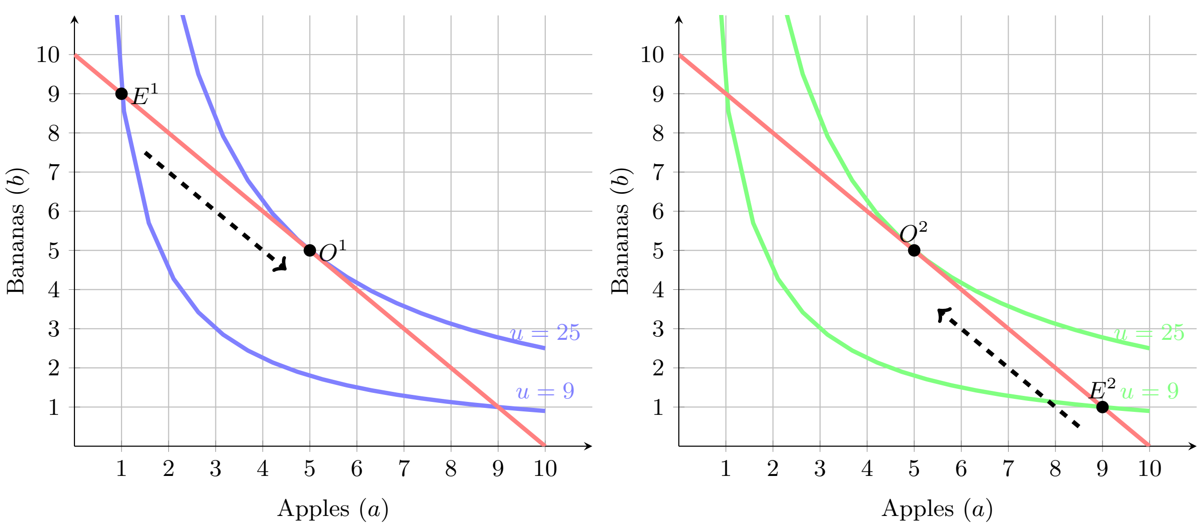


Figure 1: Endowment (E) and Optimum (O) for Person 1 (left panel) and Person 2 (right panel)

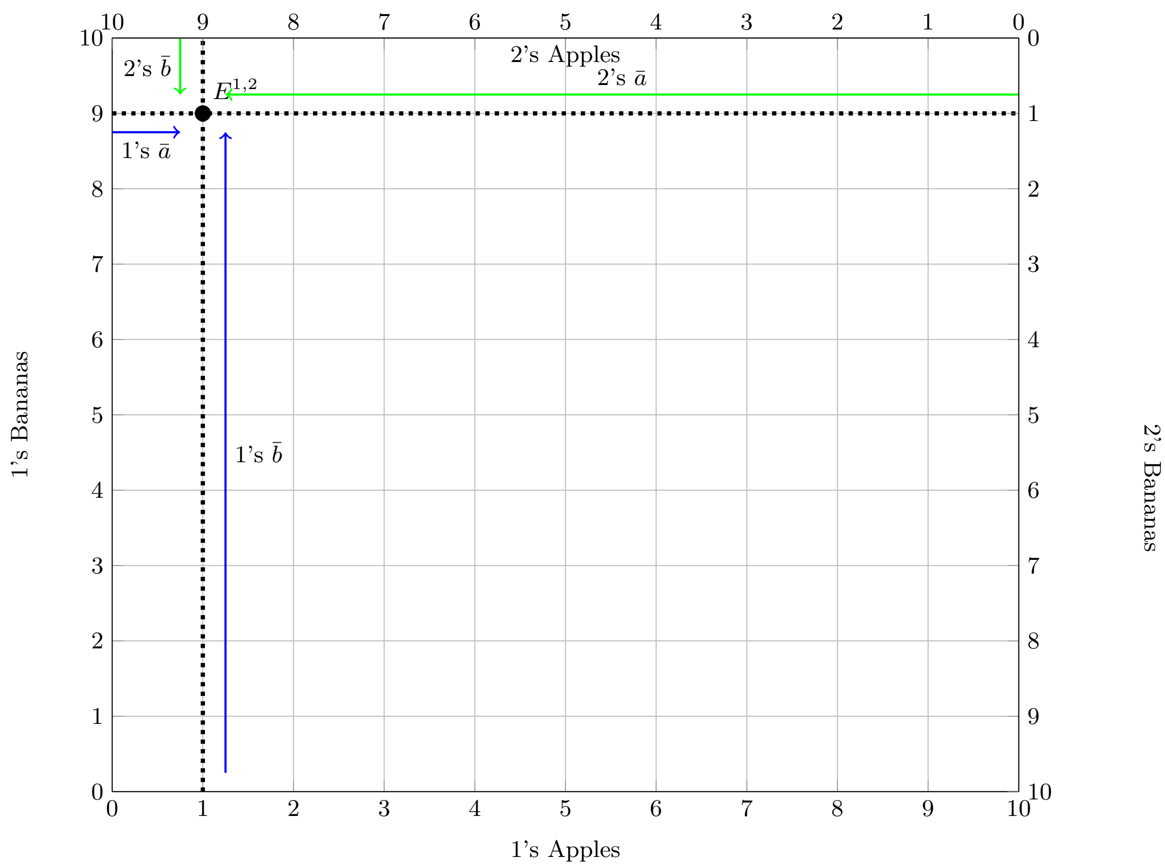
Now the ingenious tool we can use to depict both Person 1 and Person 2 at the same time is known as the **Edgeworth Box** (after economist Francis Ysidro Edgeworth). First we represent all of the resources that exist in the economy. The width of the box is the total amount of apples and the height of the box is the total amount of bananas.

Recall we have two people, person 1 has an endowment of 9 bananas and 1 apple. Person 2 has an endowment of 1 banana and 9 apples. Therefore, the "economy" consists of a total of 10 bananas and 10 apples. Thus, the height and width of the box is 10 apples and 10 bananas. Any point within the box is a **feasible allocation** of apples and bananas–that is, it allocates all 10 apples and 10 bananas somehow between person 1 and person 2. For example, person 1 could have 10 apples and 10 bananas and person 2 nothing; person 1 could have 2 apples and 8 bananas and person 2 could have 8 apples and 2 bananas, etc. All such that:

$$a^1 + a^2 = \bar{a}^1 + \bar{a}^2$$
$$b^1 + b^2 = \bar{b}^1 + \bar{b}^2$$

The left of each equation is the total amount of apples (or bananas) that each person consumes at the end, on the right of the equation is the total amount that each person starts off with (their endowments).

On the graph below, person 1 starts at the bottom left corner. Moving to the right increases the number of person 1's apples (and vice versa), moving upwards increases the number of person 2's bananas (and vice versa). Person 2 starts at the top right corner, moving to the left increases the number of person 2's apples (and vice versa) and moving downwards increases the number of person 2's bananas (and vice versa). Note that as person 1 gains more of a good, that necessarily requires person 2 to oppositely lose more of a good (and vice versa)!



Now the beauty of the Edgeworth box is that we can represent both consumers' optimization problems in the same graph, going in two different directions! I have placed several indifference curves for person 1 (in blue), indicating that that (normal) movement in the northeast direction (more apples and/or bananas for 1) increases their utility. I have also placed several indifference curves for person 2 (in green), indicating that movement in the southeast direction (more apples and/or bananas for 2) increases their utility.

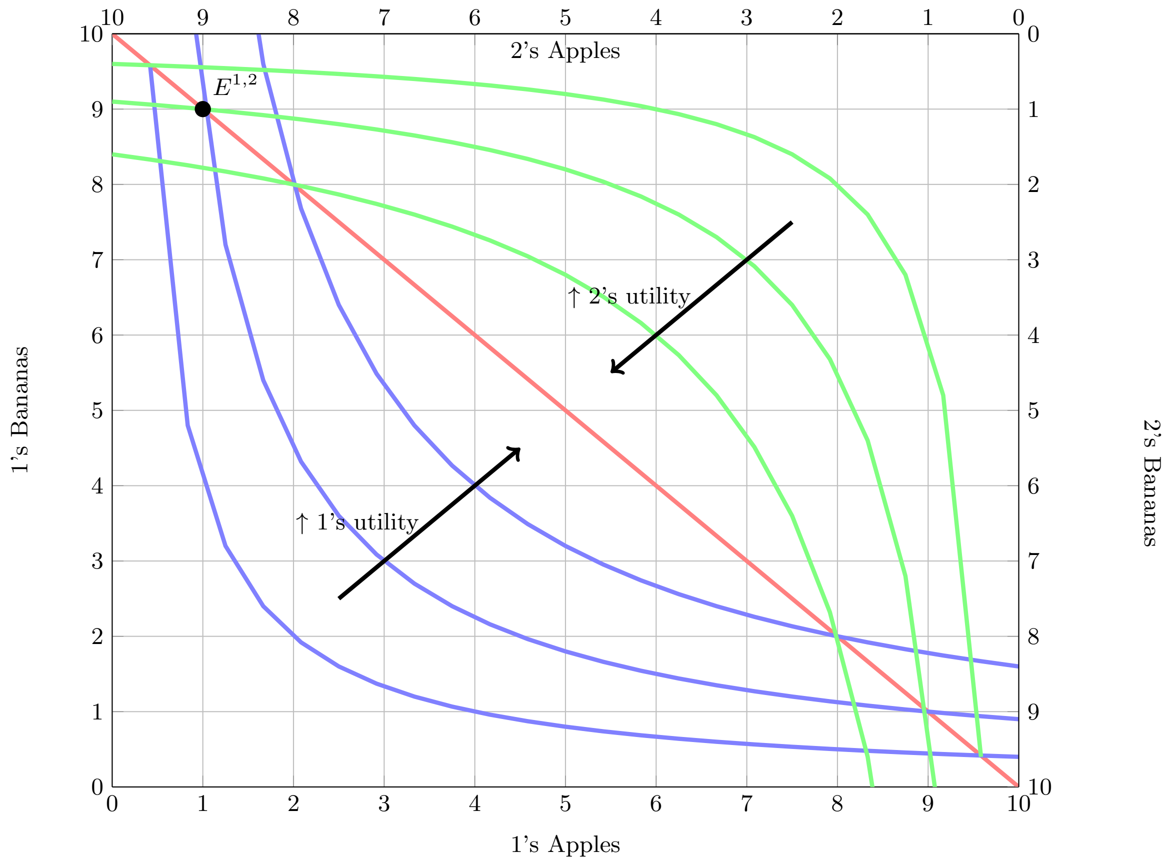


Figure 2: Edgeworth Box

Although we have defined the box such that increases to one person's consumption of a good require the decrease in the other's consumption of that good, it is a mistake to believe that exchange is a *zero-sum game*, that for one person to gain, the other must lose. Exchange is mutually beneficial between two parties when at least one of the following conditions is true

- They have different endowments
- They have different preferences
- They have different production opportunities

In this case, the two individuals have similar preferences[1] and different endowments.[2]

So let's examine what range of feasible allocations would be preferred to their original endowments by *both* parties. Recall that due to monotonicity, the areas above and to the right of their indifference curve are better. For person 1, that is movement to the NW, for person 2, that is movement to the SE. The *overlap* between these two regions is highlighted in Figure 3 below – this is the region where both parties can **gain from exchange**.

[1]They have the same utility function $u(a, b) = ab$, which is convex, indicating they both prefer an "average" bundle of some apples, some bananas, over an "extreme" bundle of mostly one good and few of another – which, in fact, is what they each start out with!

[2]We ignore production in this example, but you should have had some experience with the idea of *comparative advantage*, that individuals with different production opportunities (specifically, opportunity costs) benefit from specializing and exchanging.

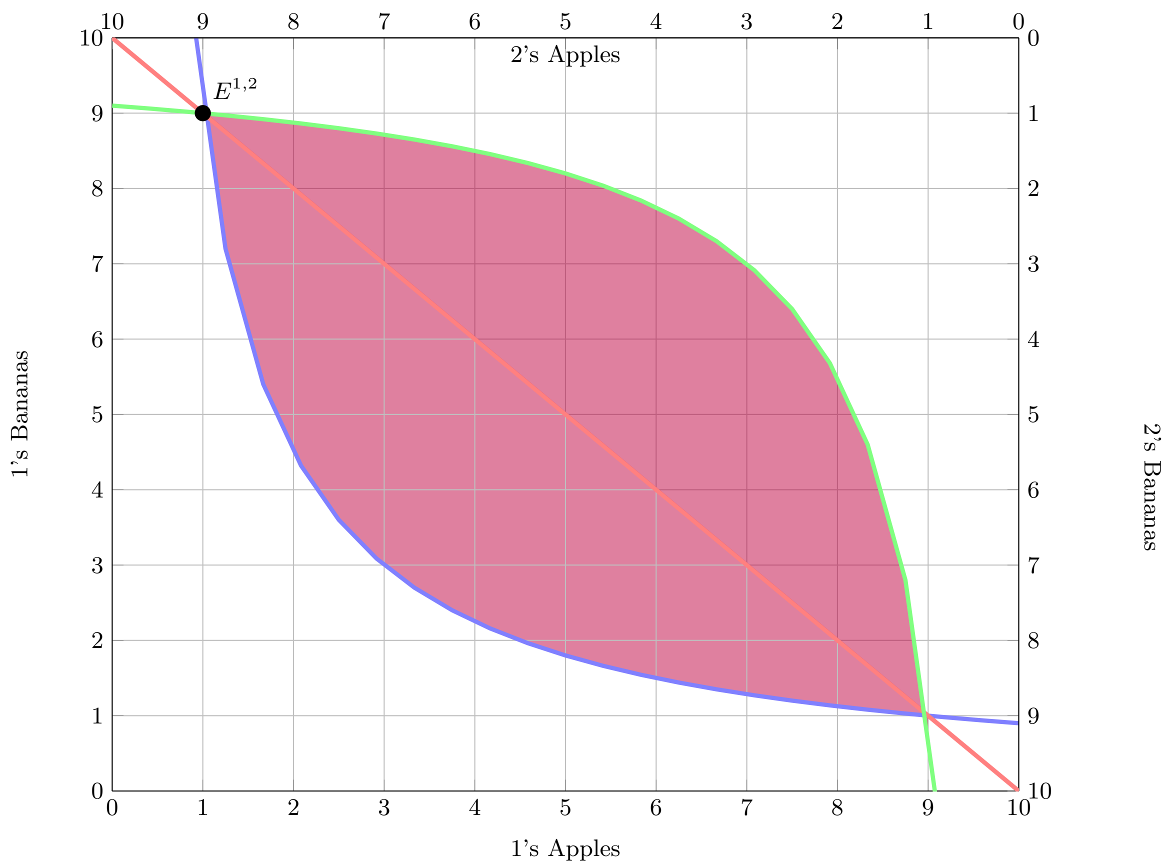


Figure 3: Edgeworth Box Showing Gains from Exchange from Endowment $E^{1,2}$

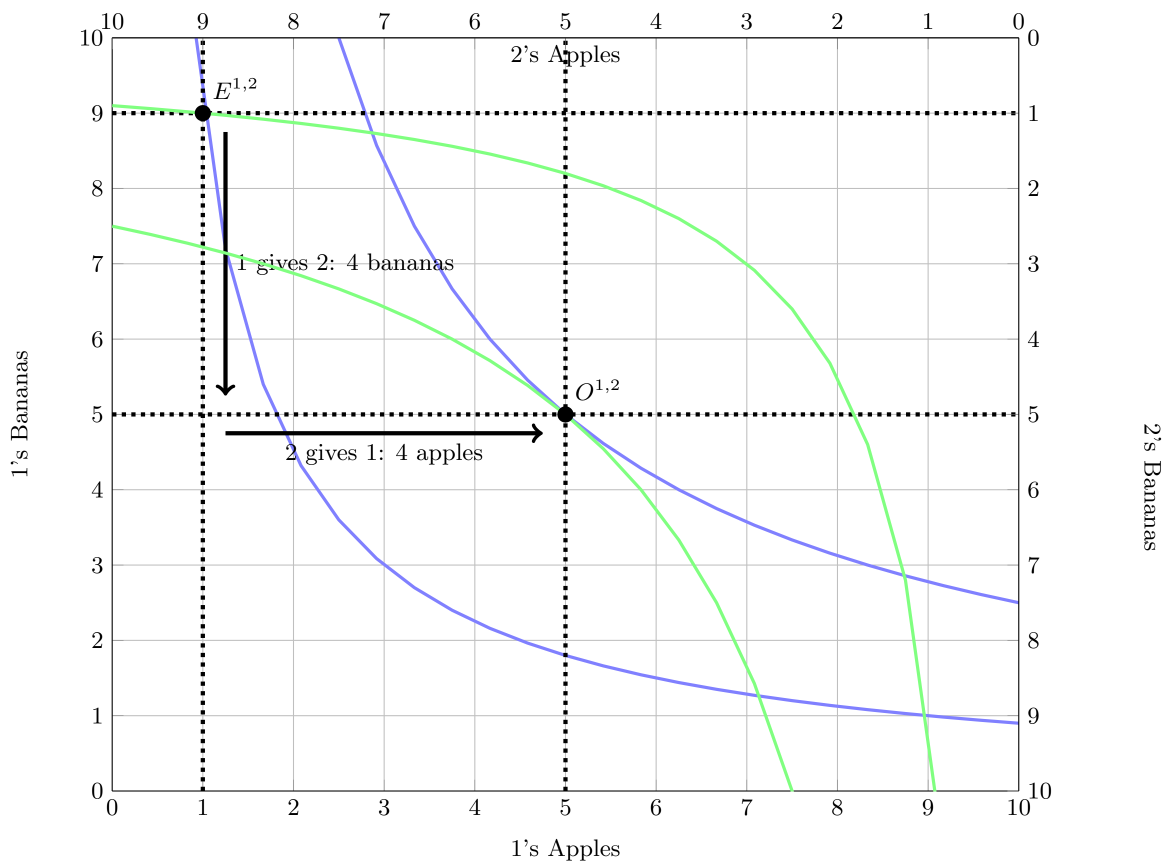


Figure 4: Edgeworth Box trade from endowment ($E^{1,2}$) to new optimum ($O^{1,2}$)

3 Pareto Efficiency

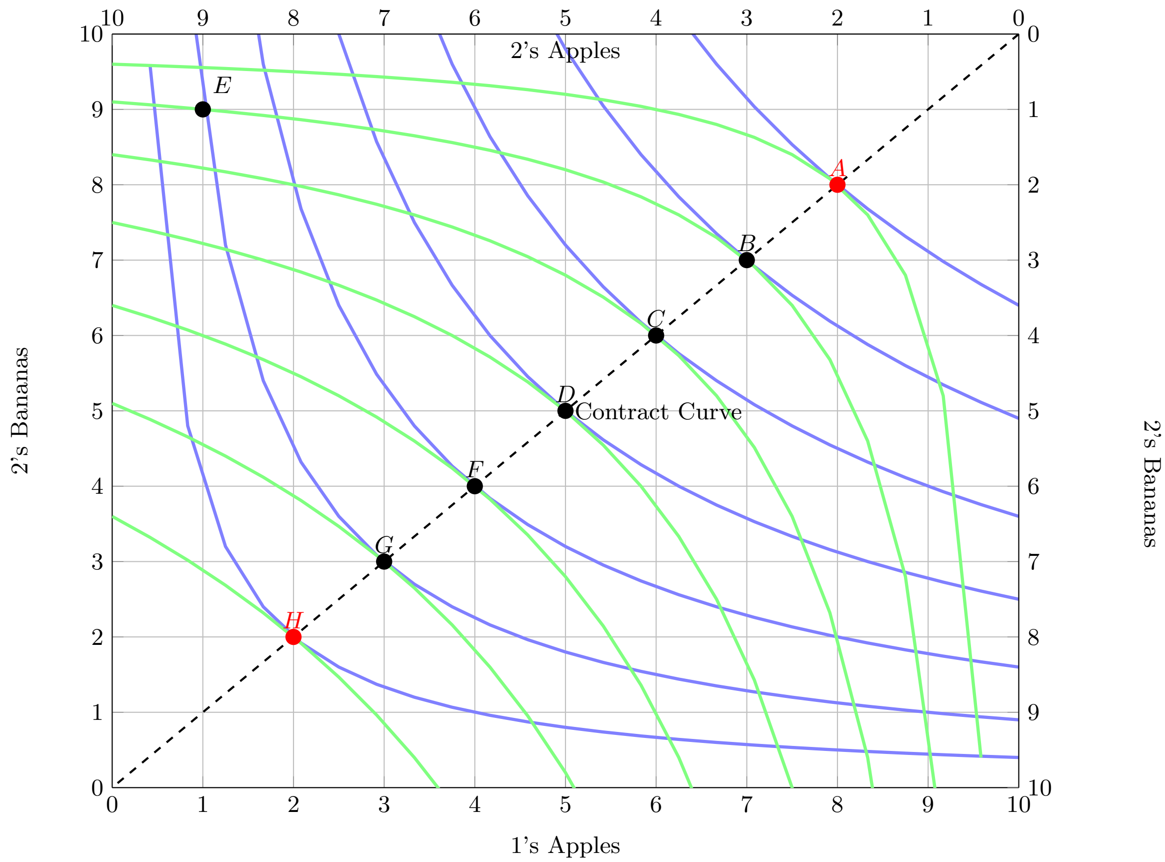


Figure 5: Edgeworth Box and Pareto Improvements from E

Any exchange is a **Pareto Improvement** if at least one party benefits and no party is made worse off. If both individuals start at endowment point E:

- $E \rightarrow H$: not a Pareto improvement (2 gains and 1 loses utility)
- $E \rightarrow G$: Pareto improvement (2 gains utility and 1 is indifferent)
- $E \rightarrow F, D, C$: Pareto improvement (both 1 and 2 gain utility)
- $E \rightarrow B$: Pareto improvement (1 gains utility and 2 is indifferent)
- $E \rightarrow A$: not a Pareto improvement (1 gains and 2 loses utility)

The set of Pareto improvements depends on the location of the endowment. If our two individuals instead start at an endowment point of J, only movements to points F, D, or C are Pareto improvements.

If there are no possible Pareto improvements, an allocation is said to be **Pareto efficient**. Suppose, starting from endowment point E, person 1 and 2 trade and end up at point F (person 1 gives 3 apples for 5 of person 2's bananas). At point F, there are no possible Pareto improvements!

This is because at any point where the two individuals' indifference curves are tangent to one another (e.g. points A, B, C, D, F, G, and H), there is no change that would benefit one party (attain a higher indifference curve) without harming another (falling to a lower indifference curve). For example, if the two were to move from F to G, person 2 would be better (get a higher green indifference curve) but person 1 would be worse (gets a lower blue indifference curve).

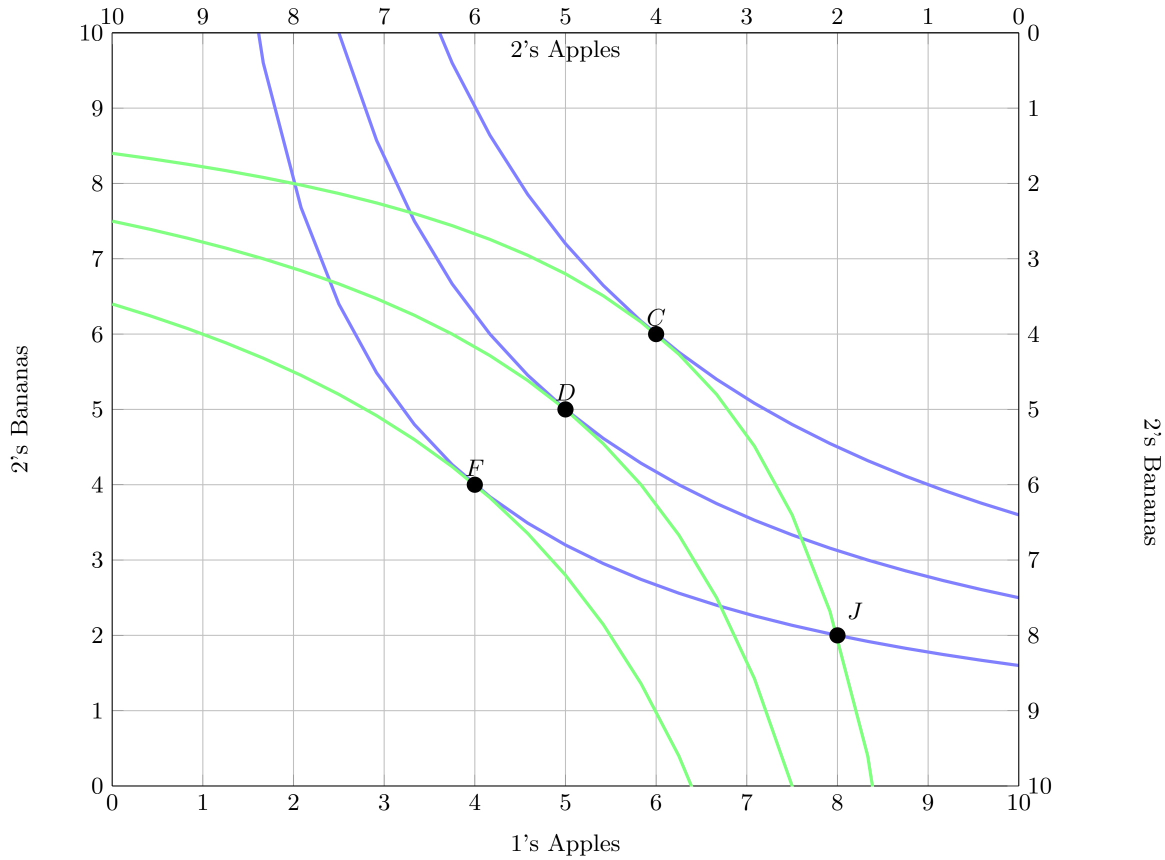


Figure 6: Edgeworth Box and Pareto Improvements from J

Note that this means there are *many* possible Pareto efficient allocations! Again, recall that the set of Pareto efficient outcomes also is affected by the endowment. Ignoring the location of the endowment, we often connect the set of Pareto equilibria with a line, known as the "contract curve," the dashed line on the graph above.

3.1 From Individual Optimum to Market Equilibrium

Because two indifference curves are tangent at a Pareto efficient outcome, this implies that the two indifference curves have the same slope, aka the Marginal Rate of Substitution is the same for both individuals. This implies that both individuals are optimizing, and if these are the only two individuals in the "market," the market tradeoff between the two goods $\frac{p_a}{p_b}$ is the same as both individual MRS's. Thus, relative prices in the market are determined by individual exchanges!

Notice that in the example above, if we pick any Pareto efficient allocation (A, B, C, D, F, G, H), the slope of the two indifference curves is always -1 (expressed in the red dashed lines in Figure below). This implies that $\frac{p_a}{p_b}$ is 1, or that the two individuals exchange at a rate of 1 apple for 1 banana. Not only have individuals found their own respective optimums, but they have together established the market price of apples and bananas through their exchanges!