Consumer Theory Application III: Consumption Tax vs. Income Tax

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Spring 2018

This handout uses consumer theory to answer a simple question: *which is better for consumers, a consumption tax (tax per unit), or an equivalent income tax that raises the same amount of revenue for the government?*

To answer this question, let's explore the following example:

Example Problem

Consumers can consume good (x) and all other goods (y) and earn utility according to:

$$\begin{aligned} U(x,y) &= xy \\ MU_x &= y \\ MU_y &= x \end{aligned}$$

A consumer has an income of \$100, and the current market price of good x is 2. Normalize the price of all other goods to \$1.

Suppose the government is considering between levying a \$3 tax on the consumption of good x or a tax on income that would raise the same amount of revenue. Are consumers better off under the good x tax or a revenue-equivalent income tax?

Pre-Tax Optimum

First let's find the consumer's optimum before the tax. At the optimum:

$$\begin{aligned} \frac{MU_x}{MU_y} &= \frac{p_x}{p_y} \\ \frac{y}{x} &= \frac{2}{1} \\ y &= 2x \end{aligned}$$

We know consumers will spend 2 times more of their income on non-good x items than on good x. To find the exact quantities, plug this equation into the budget constraint:

$$\begin{aligned} p_x x + p_y y &= m \\ 2x + y &= 100 \\ 2x + (2x) &= 100 \\ 4x &= 100 \\ x &= 25 \end{aligned}$$

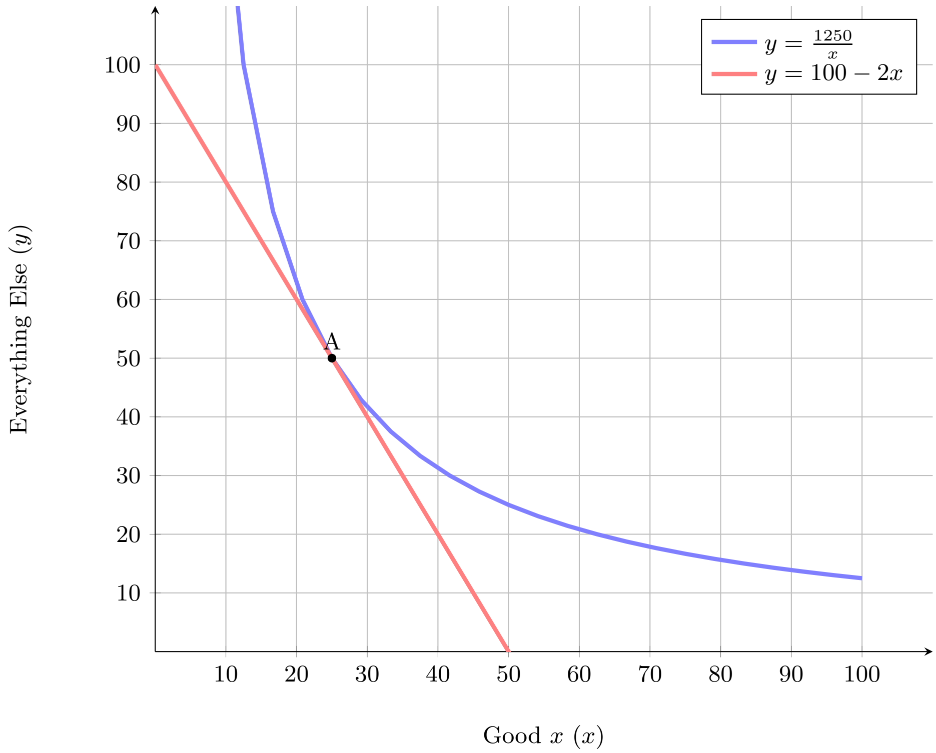

Figure 1: Consumer's optimum with no taxes

The consumer will buy 25 units of good x.

$$\begin{aligned} y &= 2x \\ y &= 2(25) \\ y &= 50 \end{aligned}$$

The consumer will spend \$50 on everything else. The consumer's total utility at this bundle is:

$$\begin{aligned} u(x, y) &= xy \\ u(x, y) &= (25)(50) \\ u &= 1,250 \end{aligned}$$

With a \$3 Consumption Tax

Now suppose the government enacts the \$3 tax on good x. The slope of the budget constraint will change from $\frac{p_x}{p_y}$ to $\frac{p_x+t}{p_y}$ or from 2 to 5.

Let's find the new optimum under the new tax. The utility function and MRS function does not change, only the prices:

$$\begin{aligned}\frac{MU_x}{MU_y} &= \frac{p_x+t}{p_y}\\ \frac{y}{x} &= \frac{2+3}{1}\\ \frac{y}{x} &= 5\\ y &= 5x\end{aligned}$$

The consumer will now spend 5 times more of their income on non-good x things than on good x. To find the new exact quantities, plug into the new budget constraint:

$$\begin{aligned}(p_x+t)x + p_y y &= m\\ (2+3)x + y &= 100\\ 5x + (5x) &= 100\\ 10x &= 100\\ x &= 10\end{aligned}$$

Now the consumer will only buy 10 units of good x.

$$\begin{aligned}y &= 5x\\ y &= 5(10)\\ y &= 50\end{aligned}$$

The consumer will still spend \$50 on everything else. The total utility the consumer will earn is:

$$\begin{aligned}u(x,y) &= xy\\ u(x,y) &= (10)(50)\\ u &= 500\end{aligned}$$

Notice the consumer is worse off than before the tax.

This tax will raise a total revenue of:

$$\begin{aligned}G &= t * x_t\\ G &= (\$3)(10)\\ G &= \$30\end{aligned}$$

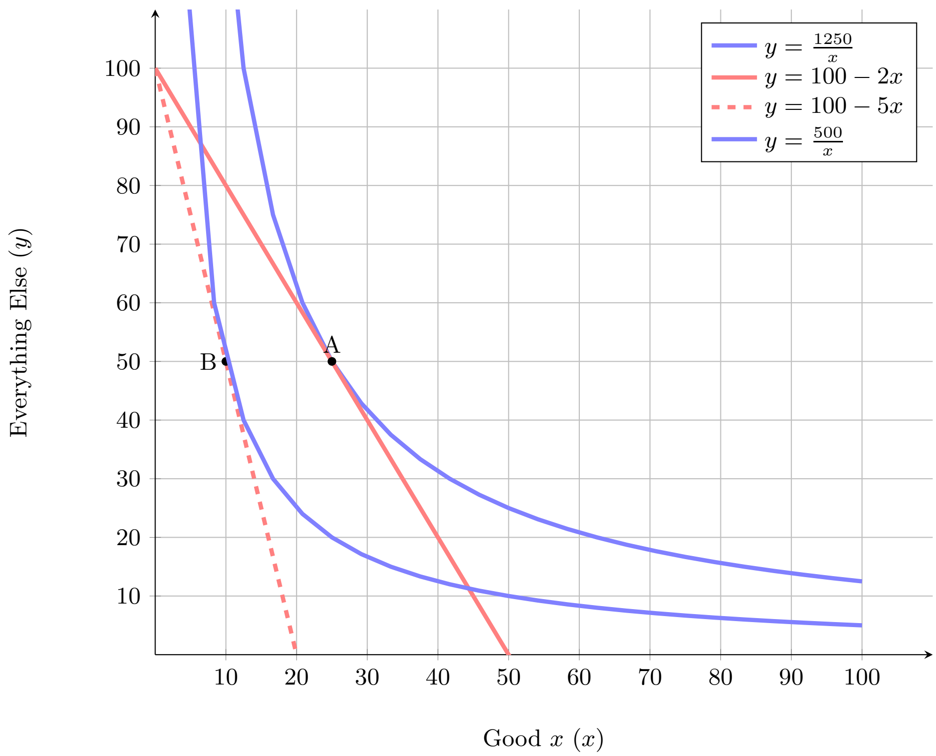

Figure 2: Consumer's optimum with no taxes (A) and with a consumption tax (B)

With an Equivalent Income Tax

If the government were to choose an income tax that raised the same amount of revenue (\$30), it would have to place a \$30 tax on income (or, since income is 100, a 30% tax). This would, instead of rotating the budget constraint by changing the relative price of good x (x), shift the original budget constraint inwards by \$30. The new budget constraint would be:

$$\begin{aligned} p_x x + p_y y &= m - 30 \\ 2x + y &= 100 - 30 \\ 2x + y &= 70 \\ y &= 70 - 2x \end{aligned}$$

Now we need to find the consumer's optimum under the income tax.

$$\begin{aligned} \frac{MU_x}{MU_y} &= \frac{p_x}{p_y} \\ \frac{y}{x} &= \frac{2}{1} \\ y &= 2x \end{aligned}$$

Notice this was the same ratio of good x to non-good x as before. But the quantities will change due to the loss of income by the income tax. Let's see the exact quantities by plugging this into the new budget constraint:

$$\begin{aligned} p_x x + p_y y &= m - 30 \\ 2x + y &= 100 - 30 \\ 2x + (2x) &= 70 \\ 4x &= 70 \\ x &= 17.5 \end{aligned}$$

The consumer buys 17.5 units of good x.

$$\begin{aligned} y &= 2x \\ y &= 2(17.5) \\ y &= 35 \end{aligned}$$

Under the income tax, the consumer earns a total utility of:

$$\begin{aligned} u(x,y) &= xy \\ u(x,y) &= (17.5)(35) \\ u &= 612.5 \end{aligned}$$

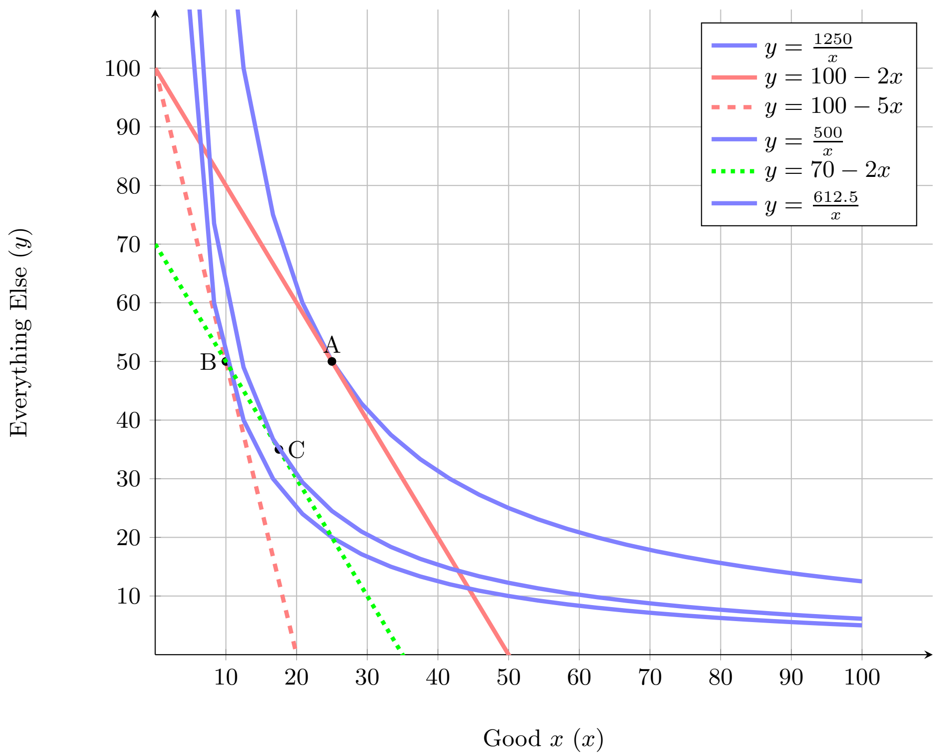

Figure 3: Consumer's optimum with no taxes (A), with consumption tax (B), and with income tax (C)

Conclusions

Scenario	Consumption of x	Consumption of y	Government Revenue	Consumer's Utility
No Taxes	25.0	50	$0	1,250.0
$3 Consumption Tax on x	10.0	50	$30	500.0
30% Income Tax	17.5	35	$30	612.5

We can see that the consumer is made worse off by either tax, but the income tax reduces the consumer's utility less than the quantity tax. Hence, the income tax is better for the consumer.